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THESIS

COMBAT SYSTEM LETHALITY

by

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generic missile system versus a sea skimming cruise missile. The second case study considers a generic medium caliber gun system versus the same sea skimming cruise missile.

Combat System Lethality

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A methodology for calculating warhead, weapon system, and combat system lethality is described for use in AE 3705, a Naval Postgraduate School course in warheads and lethality. A template to outline the methodology and two case studies using the template are developed. The case studies may be used as examples or given as class projects. The first case study considers a generic missile system versus a sea skimming cruise missile. The second case study considers a generic medium caliber gun system versus the same sea skimming cruise missile.

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I. INTRODUCTION

The Surface Warfare Officer in today's navy is faced with the substantial task of learning to operate and maintain increasingly complex combat systems. The increasing cost, complexity, and capabilities of these systems is in turn being driven by the increasing complexity and capability of opposing weapons systems that the modern combat system must be capable of handling. A key asset to any Surface Warfare Officer would be an understanding of the basic concepts and factors affecting warhead design, target survivability, and combat system lethality. In order to assist officers in developing an understanding of these concepts a course in warhead design and lethality is offered at the U. S. Naval Postgraduate School. This thesis describes a methodology for evaluating the effectiveness of a given warhead against a given target and includes two case studies in Appendix B and C which may be given to students either in the form of a class problem or as an example. A template which incorporates the methodology for working these types of problems is also provided in Appendix A to guide the student in the problem solution. The first case study considers a generic surface to air missile (SAM) system versus a generic sea skimming cruise missile. The final result of the case study is the single shot probability of kill, P_{KSS}, for the given SAM system against the given target.

The second case study evaluates the effectiveness of a medium caliber gun system using generic unguided projectiles with a proximity fuzed warhead against the same sea skimming cruise missile threat. The final result of the second case study is a cumulative probability of kill, P_K, for the gun system based on values of P_{KSS} obtained at varying ranges.

After completing two case studies involving different weapons systems against the same threat, the student should have developed an appreciation of the factors affecting each system and should be able to compare the relative effectiveness of similar combat systems.

II. WARHEAD LETHALITY

A. PROBABILITY OF A KILL GIVEN A DETONATION

A logical place to start in the determination of an overall combat systems effectiveness or lethality would be to examine the lethality of the warhead associated with the system. In the case of a proximity fuzed warhead, the most common way of measuring the warhead's lethality is the probability of a target kill given a warhead detonation, $P_{K/D}$. For a contact fuzed warhead, the lethality is measured by the probability of a target kill given a hit on the target, $P_{K/H}$. This Chapter deals with $P_{K/D}$ since most air defense systems rely on proximity fuzed warheads.

The probability of a kill given a detonation is dependent on both the design of the warhead and the target's vulnerability to the warhead damage mechanisms.

B. WARHEAD CHARACTERISTICS

Proximity fuzed warheads normally consist of a high explosive filler surrounded by a metal casing and a fuzing package. The fuzing package consists of a target detection device, TDD, a logic circuit that initiates detonation at the proper time, a safety and arming device to prevent accidental detonations, and a detonator with a primary explosive to initiate the high explosive filler. The design of the case is dependent on the type of damage mechanism desired.

High explosive, HE, warheads are classified by the type of damage mechanism they employ. The four basic types of HE warheads are blast, fragmentation, continuous rod, and shaped charge. The primary damage mechanism of a blast warhead is the shock wave produced by the high explosive

core. The case of a blast warhead is relatively thin to reduce the amount of energy lost in breaking up the case. Fragmentation warheads rely on the high velocity fragments formed when the case breaks up as their primary damage mechanism. Fragmentation warheads use either a natural fragmentation case or a controlled fragmentation case. The natural fragmentation case is a smooth case that splits into random sized fragments. The controlled fragmentation cases use scoring, grooving, external wire wrapping, or internal rings to force the case to break up into fragments of a desired size and shape. Some controlled fragmentation cases that do not require a strong case for structural reasons use preformed fragments embedded in a plastic matrix. Continuous rod warheads use a bundle of rods linked in series and placed around the outside of the warhead case, as the primary damage mechanism. The damage mechanism for shaped charge warheads is a high velocity jet and slug of molten metal formed when the shaped explosive crushes a metal liner. The fragmentation warhead is the most common type of warhead used against aircraft because it covers a large area with effective damage mechanisms. Figure 2.1 shows a typical high explosive fragmentation warhead.

The determination of $P_{K/D}$ requires the ability to calculate fragment trajectories and fragment densities at specified ranges from the burst point. A number of warhead characteristics can affect the $P_{K/D}$,including charge to metal ratio, C/M, explosive type, and fragment characteristics, such as size, weight, velocity, and total number. Another key factor is the warhead geometry which affects fragment ejection spray angles.

Factors affecting fragment trajectories are the intial fragment velocity, the static spray angles, and the fragment shape and density. Those parameters affecting fragment density are the total number of fragments, and dynamic spray angles.

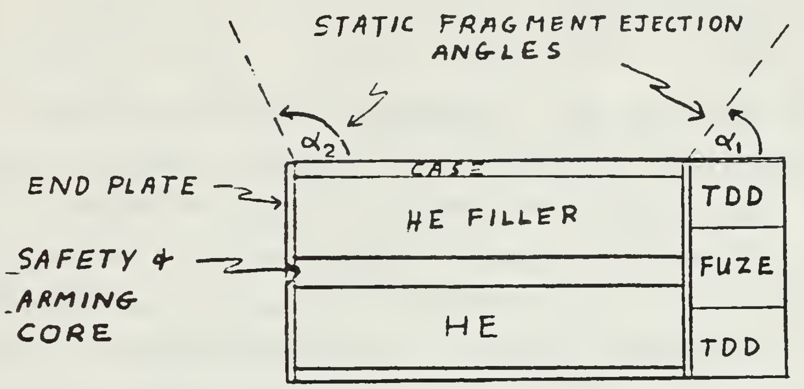


Figure 2.1 Fragmentation Warhead

1. <u>Fragment Size and Number</u>

Normally the metal case weight, M, is constrained by geometry to fit within a missile diameter or gun barrel; thus fragment size and fragment number, N, are not independent. Picking a particular fragment size specifies the total number of fragments and vice versa. The total case weight can be calculated by knowing the density of the metal and calculating the volume from the given dimensions.

$$N = \frac{\text{case weight, M, (grains)}}{\text{fragment weight (grains/fragment)}} \qquad (2.1)$$

2. <u>Fragment Initial Velocity</u>

R. W. Gurney theoretically established the initial velocity of fragments from fragmenting munitions in 1943.

He derived what is now known as the Gurney formula,

$$V_0 = \sqrt{(2E)} \sqrt{\frac{C/M}{1 + C/2M}} \qquad (2.2)$$

where V_0 is the initial fragment velocity from a static warhead detonation, $\sqrt{2E}$ is the Gurney constant which is a function of the explosive used, C is the mass of the HE charge used, and M is the total mass of the fragments. Values for the Gurney constant can be found in **[Ref. 1: Table I]**. The charge mass, C, can be determined either by calculating the volume and multiplying by the density of the explosive, or for an existing warhead by looking it up in the appropriate technical manual.

3. Fragment Density at Impact

Fragment density , ρ , is directly proportional to the number of fragments and inversly proportional to the square of the radius, R, that the fragments have traveled from the point of detonation. It is also dependent on the dynamic spray angles, ϕ_i, of the warhead. Equation 2.3 gives the fragment density as a function of R, where ϕ_1 is the dynamic fragment spray ejection angle for the leading edge of the fragments, and ϕ_2 is the trailing edge dynamic fragment spray ejection angle.

$$\rho(R) = \frac{N}{2\pi \times R^2 \times (\cos(\phi_1) - \cos(\phi_2))} \qquad (2.3)$$

Note that the R in equation 2.3 is not necessarily the distance between the warhead and target at detonation, R_{DET}, but is the actual distance the fragments have traveled at impact.

C. TARGET VULNERABILITY ASSESMENT

When calculating $P_{K/D}$ target characteristics such as target vulnerability must be considered. Target vulnerability is defined as the inability of the target to withstand one or more hits by the damage mechanisms from the warhead. Targets that are more vulnerable are softer and easier to kill. In order to determine the effect of target vulnerability on $P_{K/D}$, a vulnerability assesment must be conducted. A vulnerability assessment can be carried out at one of three general levels of detail. The three levels , in order of increasing complexity, are estimates, evaluations , and analyses. For example, an estimate will use simple equations for target vulnerability measures that are functions of a few major parameters of the target, the damage mechanisms, and the terminal effects parameters. Analyses are very detailed studies using specific technical information and are usually conducted with digital computers using complex models of the aircraft.

No matter what the level of assesment, there are certain required elements common to all levels. These common elements are; (1) a selection of target kill levels required, (2) assembly of a technical description of the target, (3) determination of the target's critical components, (4) selection of the specific weapon system to be used against the target, (5) an analysis to identify the type and amount of damage required to kill each critical component, and (6) the computation of the appropriate vulnerability measure for the components and the entire aircraft based on the weapon system chosen. An in-depth description of these six elements can be found in [Ref. 2], a summary of each element is given below.

1. Select Kill Level Required

Four different attrition kill levels have been defined with their primary difference being the amount of time it takes for the target aircraft to fall out of control. The four levels are:

-"KK"-Kill - immediate or catastrophic kill (less than 5 sec.)
-"K"- Kill _ loss of control within 30 sec.
-"A"- Kill - loss of control within 5 min.
-"B"- Kill - loss of control within 30 min..

In a shipboard air defense problem, the key factors affecting the choice of kill levels are target speed or closing rate, and weapon intercept range from own ship. In most cases the "B"-kill is not satisfactory for the air defence problem because the target could still complete its mission and re-engagement would be necessary. Even the "A"-kill may be unsatisfactory for high speed targets engaged by close in weapons systems.

2. Technical / Functional Target Description

A technical and functional description of the target must be assembled. In many cases this information may come from intelligence estimates and comparisons with similar systems. The technical description should include location, size, construction, materials used, and operation of all major subsystems and components. The functional description should define functions of these components. Cross section drawings may be all that is required for an evaluation, but a thorough assesment for an analysis requires much more.

3. Critical Component Analysis

A critical component, as defined by [Ref. 2], is any component which if damaged or destroyed would yield a

defined aircraft kill level. Some components could be critical components for an "A"-kill but not for a "KK"-kill. The first step in a critical component analysis is to identify the flight and mission essential functions that the aircraft must perform to accomplish its mission. The second step is to identify the major system and subsystems that perform these essential functions. The third step is to conduct a Failure Mode and Effects Analysis (FMEA) to identify the relationship between component failure modes and performance of the essential functions. The fourth step is to relate component failure modes to combat caused damage. This is known as the Damage Modes and Effects Analysis, or DMEA. Finally a visual presentation of the critical components, known as the kill tree, or a logical expression known as the kill expression, is developed. The kill tree / kill expression help to identify the critical components for a given kill level including any redundancies involved.

4. Damage Mechanism Selection

This consists of selecting the specific damage mechanism to be studied. Typical damage mechanisms considered are (1) non explosive penetrators or fragments, (2) fragments and blast from internally detonating warheads, (3) external blast, (4) the fragments, penetrators, and debris from externally detonating warheads.

5. Critical Component Kill Criteria

Once the critical components have been identified the kill criteria for each of the failure modes of the components must be determined for each of the selected threats. Four specific kill criteria are currently used for externally detonating warheads. They are the probability of a component kill given a hit, $P_{k/h}$, the area removal criterion, the energy density criterion, and the blast damage criterion.

a. The $P_{k/h}$ Function

The $P_{k/h}$ function defines the probability of a component kill when hit by a fragment or penetrator. This criterion is a function of the mass and velocity of the damage mechanism. The determination of the $P_{k/h}$ for a given component is extremely difficult. Numbers for $P_{k/h}$ are eventually assigned based upon a combination of empirical information, engineering judgement, experience with similar systems, and experimental or combat data. Although values of $P_{k/h}$ for individual components may not be precise, they can be quite useful when conducting comparisons between different damage mechanisms. The $P_{k/h}$ criterion is particularly useful against components that can be killed with a single hit such as hydraulic lines, electronics, crew members, control rods, fuel tanks, warheads, and engines. Components where the $P_{k/h}$ function would not be as useful are large area type components such as wings or control surfaces.

b. Area Removal

The area removal criterion defines a specific amount of area that must be removed from a component in order to kill that component. This criterion is particularly useful in evaluating the effect of large penetrators or closely spaced fragments against structural components, or lifting and control surfaces.

c. Energy Density

A component kill is expressed in terms of a minimum component surface area exposed to a threshold level of kinetic energy density. It is applicable to closely spaced fragment hits on structural components, control surfaces, and large components such as fuel tanks or engines.

d. Blast

A component kill is usually expressed in terms of a minimum overpressure on an aircraft surface necessary to cause a component kill for the specified kill level. This criterion is usually applied to structural components and lifting / control surfaces.

6. Computation of Vulnerability Measures

a. Vulnerability to Non-explosive Penetrators or Fragments

An aircraft's vulnerability to penetrators or fragments can best be measured by its vulnerable area, A_V. The vulnerable area concept can be applied at the component level and is defined as the product of the component's total presented area, A_p, and the component's $P_{k/h}$. For the ith component,

$$A_{v_i} = A_{p_i} \times P_{k/h_i} \tag{2.4}$$

Since both presented area and $P_{k/h}$ vary with aspect, the vulnerable area also varies with aspect. For an aircraft with nonredundant critical components and no critical component overlap, the sum of the vulnerable areas is equal to the total vulnerable area of the aircraft

$$A_V = \sum A_{v_i} \tag{2.5}$$

The probability of an aircraft kill given a hit on the aircraft is

$$P_{K/H} = A_V / A_P . \tag{2.6}$$

Cases where critical components overlap each other complicate the computation of vulnerable area. In the case of overlap, the overlap area is normally considered as a separate component and separate $P_{k/h}$ and A_v are computed for the overlap region. In computing the $P_{k/h}$ for the overlap area, the fact that kills of more than one component are not mutually exclusive must be considered. The $P_{k/h}$ for the overlap area becomes;

$$P_{k/h_o} = 1 - P_{S_o} = 1 - (1 - P_{k/h_1})(1 - P_{k/h_2}) \qquad (2.7)$$

for two critical components, andthe vulnerable area is

$$A_{v_o} = A_{P_o} \times P_{k/h_o} \qquad (2.8)$$

Provided the individual components $P_{k/h}$ remains the same, overlapping components result in a reduced vulnerable area. Sometimes overlapping components can change the $P_{k/h}$ of the individual components. For example, overlapping a fuel tank and engine can lead to increased $P_{k/h}$ for the engine resulting in an overall increase in vulnerable area for the overlap region.

Cases where there are redundant components or redundant components with overlap further complicate the measurement of vulnerability. A more in depth coverage of these situations may be found in [Ref. 2].

b. Vulnerability To Externally Detonating HE Warheads

The primary damage mechanisms associated with externally detonating warheads are the fragments from the warhead case. Blast from the high explosive is usually a secondary effect. In most cases a warhead detonation close enough to kill a target by blast will also kill the target

with fragments. As in the single fragment case, vulnerable area is still an intrinsic measure of a targets vulnerability to an externally detonating warhead.

Another more descriptive measure of vulnerability is the probability of a kill given a detonation of the external warhead, $P_{K/D}$. $P_{K/D}$ is equal to $\bar{P}_K^{(n)}$, the probability the aircraft is killed by n independent, random hits from the detonation. In general,

$$P_{K/D} = \bar{P}_K^{(n)} = 1 - \prod_{j=1}^{n}(1 - P_{K/H}^{(j)}) \tag{2.9}$$

where $P_{K/H}^{(j)}$ is the probability the aircraft is killed by the jth random hit, and n can be calculated by the product of the fragment density, ρ, and the area presented, A_P.

$$n = \rho \times A_P \tag{2.10}$$

For small $P_{K/H}^{(j)}$,

$$\prod_{j=1}^{n}(1 - P_{K/H}^{(j)}) = \exp(-\sum_{j=1}^{n} P_{K/H}^{(j)})$$

and since,

$$P_{K/H}^{(j)} = A_V^{(j)} / A_P$$

and

$$P_{K/D} \cong 1 - \exp(-\rho/n \sum_{j=1}^{n} A_V^{(j)})$$

For the case of no redundant critical components, the vulnerable area for the jth hit, $A_V^{(j)}$, is assumed to be constant for all hits, leaving

$$P_{K/D} \cong 1 - \exp(-\rho A_V). \tag{2.11}$$

D. DETERMINATION OF $P_{K/D}$: THE ENDGAME ANALYSIS

In order to determine the relationships between the variables within the $P_{K/D}$ expression, an "Endgame" analysis must be conducted. Equation 2.11 gives an approximation for $P_{K/D}$. In order to evaluate this expression for a detonation at a given point, the fragment density and the target vulnerable area must be known. Equation 2.3 gives us an expression for ρ as a function of distance from the blast, R, the dynamic spray angles ϕ_1 and ϕ_2, as well as the number of fragments, N. The dynamic spray angles are functions of both static and dynamic warhead parameters, as well as target speed, and encounter geometry. A convenient distance to use in $P_{K/D}$ calculations is the distance from the target that the detonation takes place, R_{DET}. Since the target moves during the time between warhead detonation and fragment impact R is not equal to R_{DET}, and a relationship between the two must be developed. Another factor to be considered in the determination of $P_{K/D}$ is that the vulnerable area changes with aspect, hence the average impact angle of the fragments with respect to the target is needed.

1. Early vs. Late Bird

The first step in the endgame analysis is to determine the relative geometry involved in the warhead / target encounter. There are two basic types of intercept geometry that can occur, the "Early" bird and the "Late" bird. In the early bird geometry the warhead crosses the target line of flight ahead of the target and in the late bird the warhead crosses behind the target. The primary difference between the two geometries is that the sign of the warheads static fragment ejection spray angles change. For fragments coming off the top of the warhead the static ejection angles are positive and for fragments coming off the bottom of the

warhead the static ejection angles are negative. The assumption of early or late bird geometry must be made before proceeding with the endgame analysis.

2. Dynamic Spray Angles

The dynamic spray angles, ϕ_i, are determined from the vector addition of the missile velocity vector, $\vec{V}_M$, the average fragment velocity vector, $\vec{V}_o$, and the reversed target horizontal velocity vector, $\vec{V}_T$. The dynamic spray angle, ϕ_i, is then found by taking the arctangent of the ratio of the X and Y components of the resultant vector which gives a dynamic angle relative to the horizontal axis, γ_i, and then subtracting the missile heading, θ, to get a spray angle relative to the missile axis. Equation 2.12 and 2.13 give the dynamic spray angles.

$$\phi_i = \gamma_i - \theta \ , \quad \text{for } i = 1,2 \tag{2.12}$$

$$\gamma_i = \text{Arctan}\left(\frac{V_m \sin\theta + V_o \sin(\theta + \alpha_i)}{V_m \cos\theta + V_o \cos(\theta + \alpha_i) - V_T}\right) \tag{2.13}$$

Another useful angle to determine at this time is the angle of the center fragment with respect to the reference direction, γ_{CF}. This is done by taking the average of the γ_i as shown in equation 2.14.

$$\gamma_{CF} = (\gamma_1 + \gamma_2)/2 \tag{2.14}$$

This is useful because the reference direction is normally picked to be in line with the target's course, which makes γ_{CF} the fragment incident angle on the target. Figure 2.2 is a diagram of the endgame parameters.

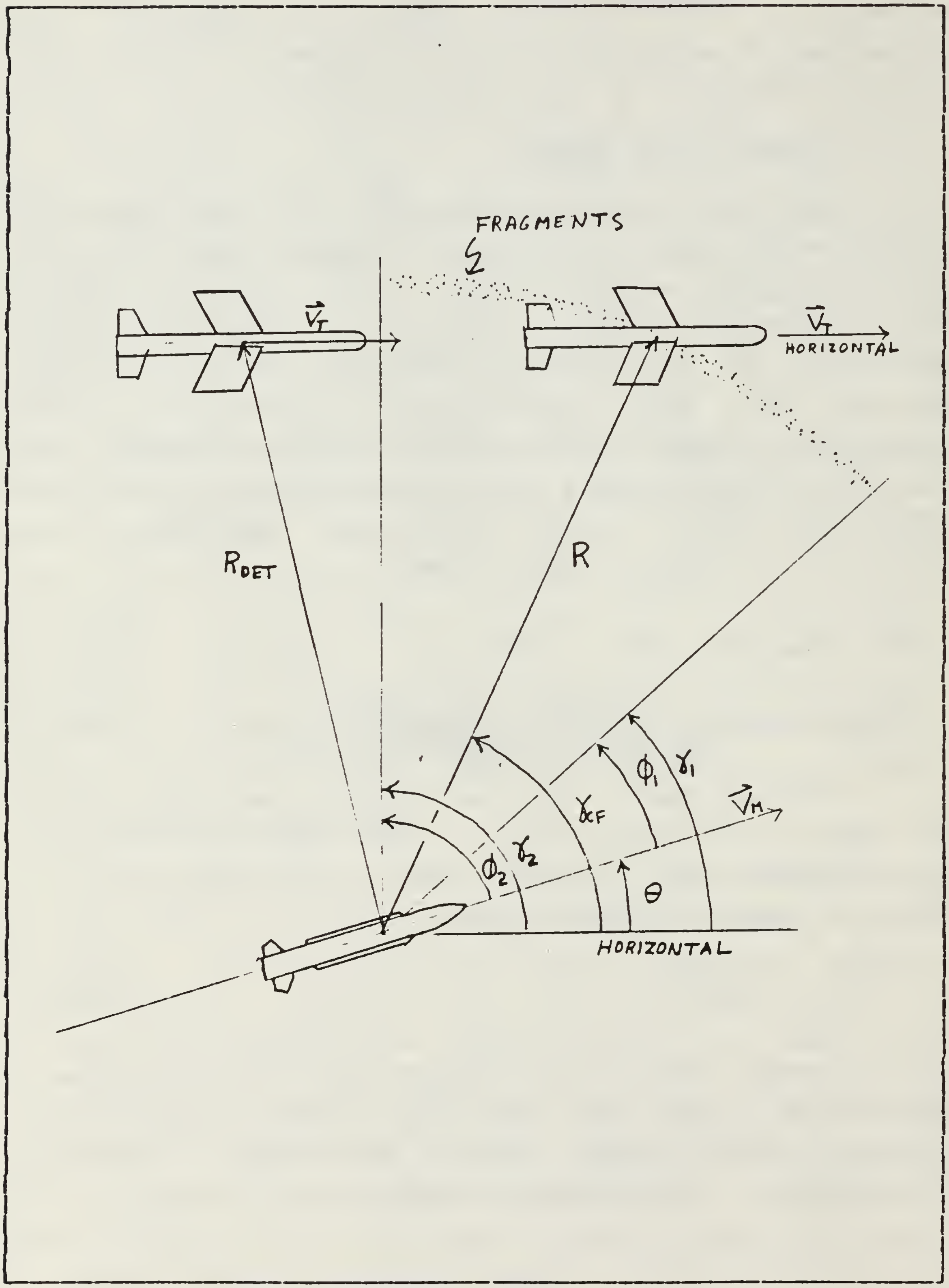


Figure 2.2 Endgame Parameters

3. Detonation Range vs. Fragment Travel Range

The ratio of R to R_{DET} is needed so that the fragment spray density, ρ, in the $P_{K/D}$ equation can be described in terms of R_{DET}. The ratio is determined by taking the ratio of the magnitudes of the absolute fragment velocity vector, $\vec{V}_{fa}$, and the fragment velocity vector relative to the target, $\vec{V}_{f/T}$. Equations 2.15 through 2.17 give the ratio between R and R_{DET}.

$$\frac{R}{R_{DET}} = \frac{|\vec{V}_{fa}|}{|\vec{V}_{f/T}|} = \frac{|\vec{V}_o + \vec{V}_M|}{|\vec{V}_o + \vec{V}_M - \vec{V}_T|} \tag{2.15}$$

where

$$|\vec{V}_{fa}| = V_{fa} = \sqrt{(V_m + V_o \cos\alpha_{CF})^2 + (V_o \sin\alpha_{CF})^2} \tag{2.16}$$

and

$$|\vec{V}_{f/T}| = \sqrt{(V_{fa} \cos(\gamma_{CF}) - V_T)^2 + (V_{fa} \sin\alpha_{CF})^2} \tag{2.17}$$

4. Effect of Fragment Incident Angles on Vulnerable Area

In a vulnerability analysis the vulnerable area is determined from many aspects. The A_V from the aspect closest to the fragment incident angle is used in the calculation of $P_{K/D}$. In a vulnerability estimate the vulnerable area is computed for only a few aspects. If the fragment incident angle is not close to one of these aspects an approximation must be made.

A simple approximation to the effect of fragment incident angle on vulnerable area is to project the vulnerable area from two orthogonal target aspects onto a plane perpendicular to the fragment incident angle. This is called the projected vulnerable area and is given by equation 2.18.

$$\text{Projected } A_V = A_{V_1} \left|\sin(\gamma_{CF})\right| + A_{V_2} \left|\cos(\gamma_{CF})\right| \qquad (2.18)$$

Figure 2.3 shows the geometry involved.

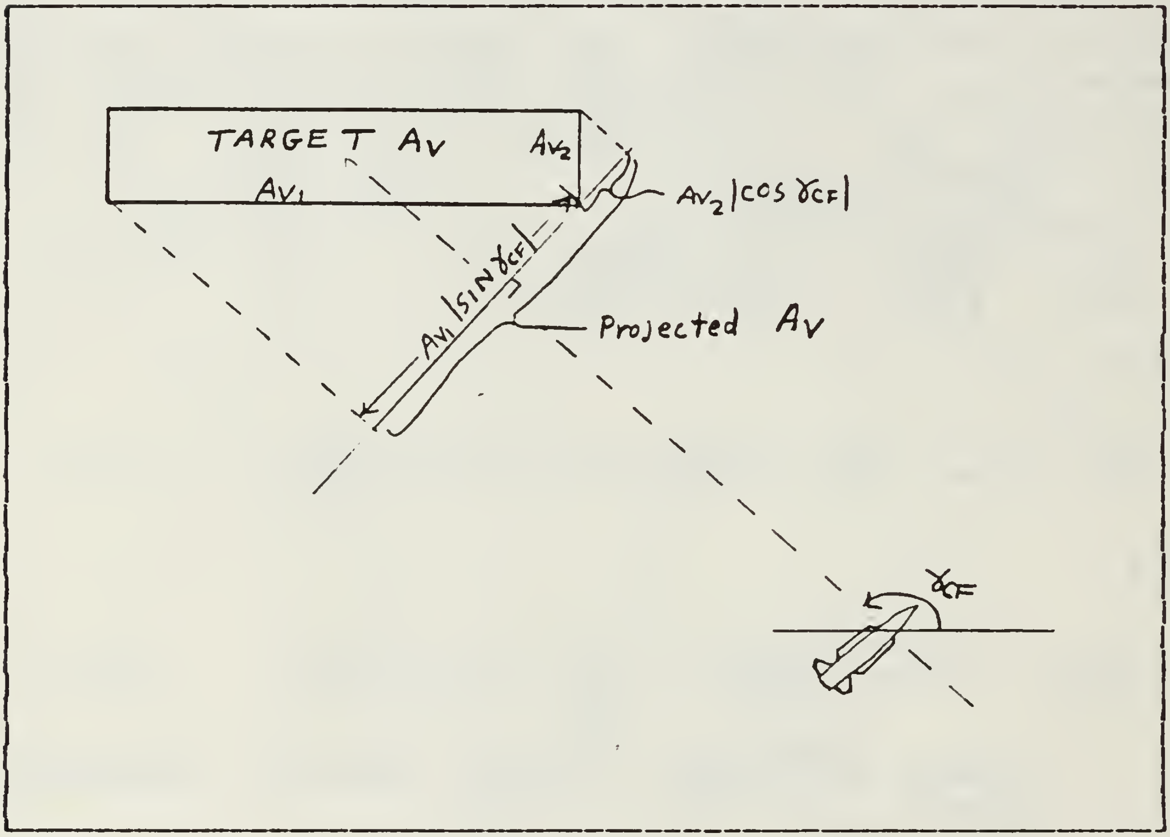


Figure 2.3 Projected Vulnerable Area

5. <u>Calculation of $P_{K/D}$</u>:

Having determined the dynamic spray angles, ϕ_i, the ratio of R to R_{DET}, and the projected A_V we can now determine the fragment spray density as a function of detonation range, $\rho(R_{DET})$, and $P_{K/D}$ as a function of detonation range, $P_{K/D}(R_{DET})$, from equations 2.15 and 2.3. Equation 2.19 gives the fragment density as a function of detonation range, and equation 2.20 gives us the $P_{K/D}$ as a function of R_{DET}.

$$\rho(R_{DET}) = \frac{N}{2\pi \times (R_{DET} \times R/R_{DET})^2 \times (\cos\phi_1 - \cos\phi_2)} \qquad (2.19)$$

$$P_{K/D}(R_{DET}) = 1 - \exp(-\rho(R_{DET}) \times \text{projected } A_V) \qquad (2.20)$$

In calculating this $P_{K/D}$, the assumption is made that the fragment spray pattern covers the entire target. Figure 2.4 shows the dynamic spray angles superimposed on a target. The spray angles define four separate zones where fragments hit the target. If a warhead detonates in zone I, all of the fragments will cover only a portion of the target. In zones II and III a portion of the spray pattern will cover a portion of the target. Only in zone IV does all of the spray cover the entire target. In zones I,II, and III prior to calculating $P_{K/D}$, the vulnerable area of the target will have to be reduced to reflect the actual area covered by the spray pattern.

6. Lethal Radius

Since $P_{K/D}$ is highly dependent on the point of detonation, another, sometimes more useful, measure of target vulnerability to a given warhead is the lethal radius, r_ℓ. The lethal radius is determined by finding the point along the centerline of the fragment spray zone, γ_{CF}, where the $P_{K/D}$ equals one half. Figure 2.5 shows a typical $P_{K/D}$ function and the lethal radius. The lethal radius parameter is useful for evaluating different warheads against the same target, and also is used in the calculation of P_{KSS}.

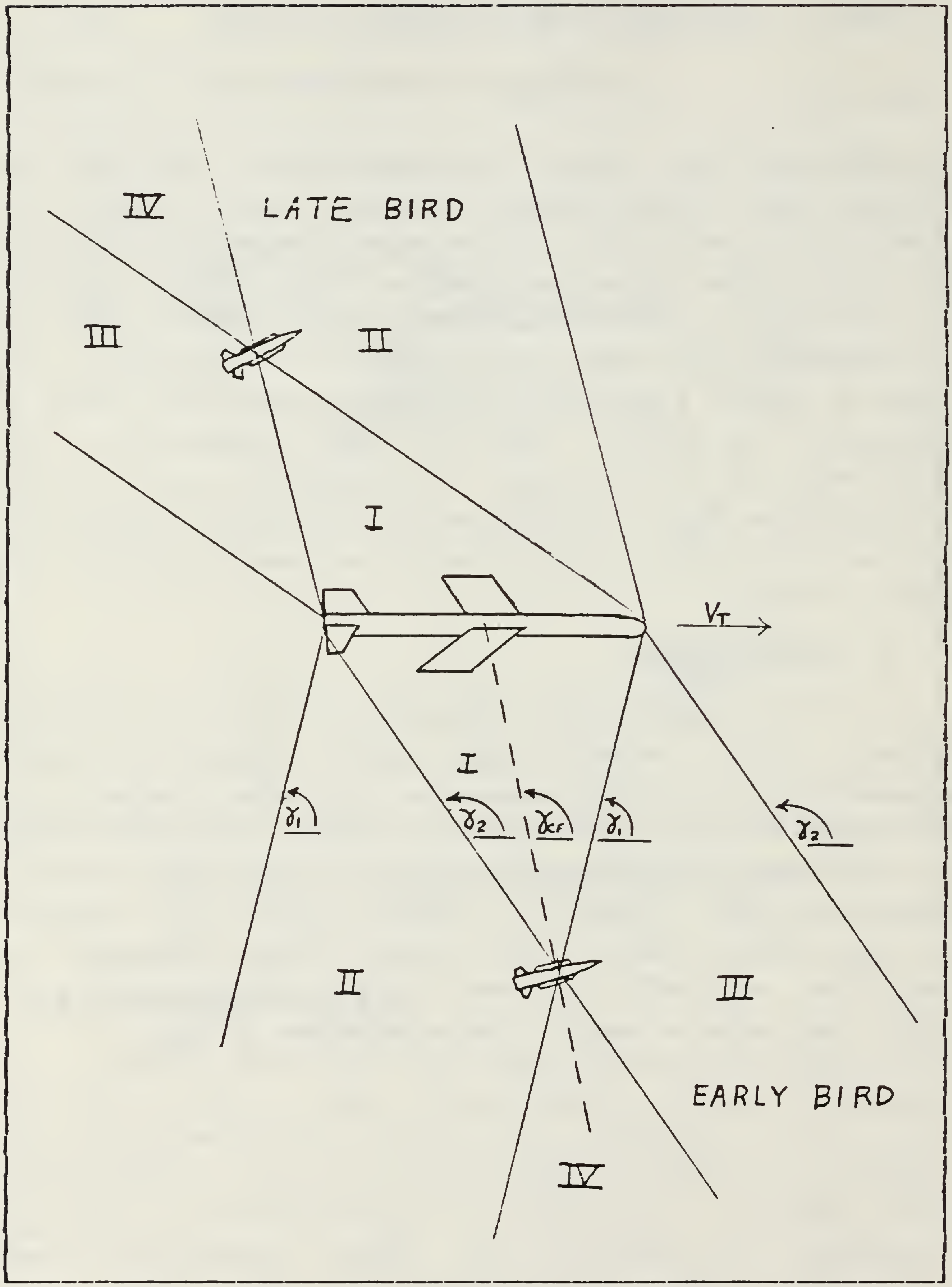


Figure 2.4 Fragment Spray Zones

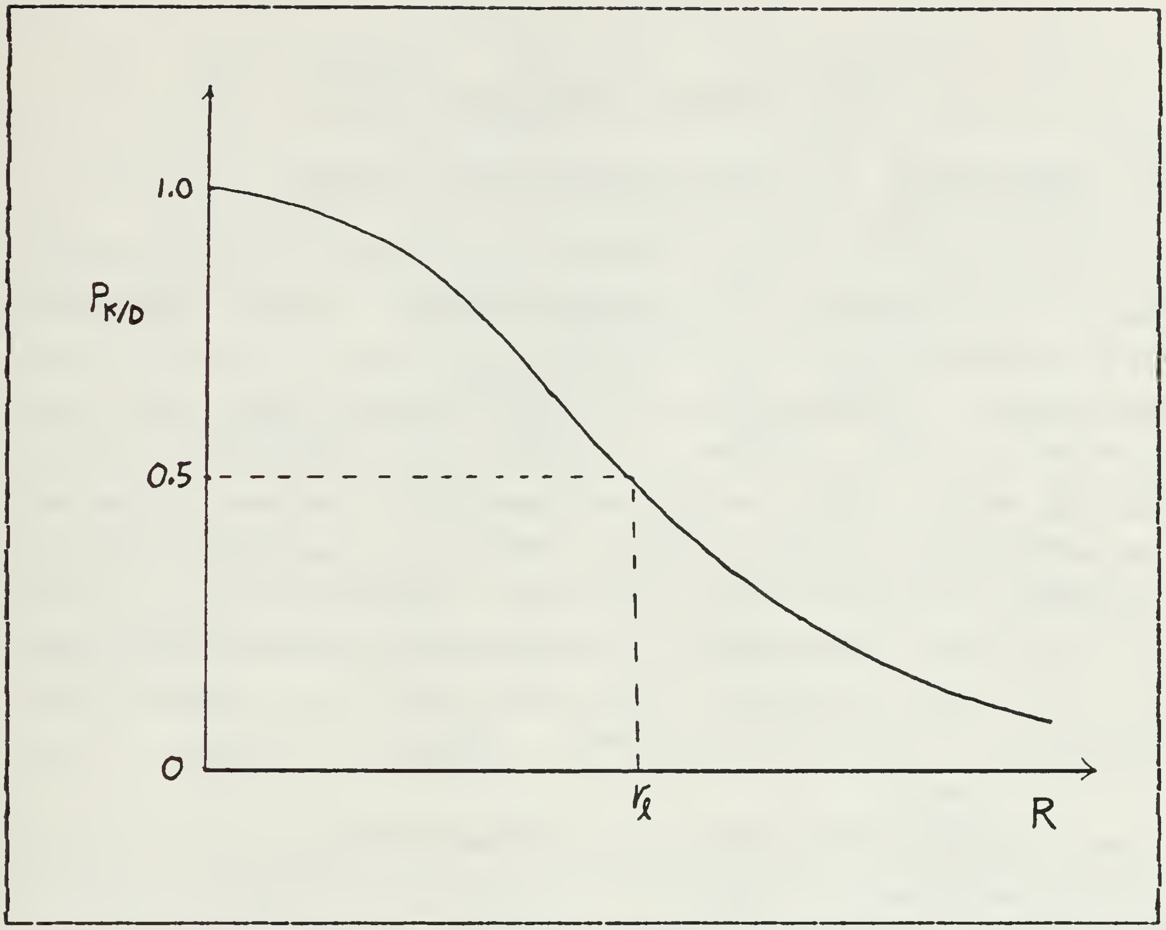


Figure 2.5 Lethal Radius

III. WEAPON SYSTEM LETHALITY

A. PROBABILITY OF A KILL GIVEN A SINGLE SHOT

The probability of an aircraft kill given a single shot, P_{KSS}, is a measure of a weapons systems overall effectiveness or lethality. P_{KSS} is computed on the assumption that the target has been detected and that a weapon has been launched or fired. Thus, the P_{KSS} takes into account the ability of the fire control and guidance systems to direct the propagator to the target's vicinity, the probability of correctly fuzing, and the target vulnerability to the warhead damage mechanisms. A P_{KSS} can be computed for both gun and missile systems using the basic two dimmensional equation in the intercept plane.

$$P_{KSS} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho(x,y) * P_f(x,y) * V(x,y)\, dx\, dy \qquad (3.1)$$

Equation 3.1 shows how the P_{KSS} is calculated, where $\rho(x,y)$ is the miss distance frequency distribution, $P_f(x,y)$ is the probability of fuzing for a HE/FRAG warhead, and $V(x,y)$ is the kill function that defines the probability the target is killed due to a propagator whose trajectory intersects the intercept plane at x,y. For a contact fuzed warhead, the kill function $V(x,y)$ is simply the probability of a kill given a hit, $P_{K/H}(x,y)$, over the target area and is zero elsewhere. For the proximity fuzed warhead, $V(x,y)$ becomes the probability of a kill given a warhead detonation, $P_{K/D}$, which was covered in Chapter two.

B. MISS DISTANCE DISTRIBUTION FUNCTION

The miss distance frequency distribution function, $\rho(x,y)$ or $\rho(r)$, can be thought of as a measure of the weapon systems fire control or guidance accuracy. Chapter six of [Ref. 2] expresses $\rho(x,y)$ as the bivariate normal distribution shown below in equation 3.2, where and are the means, and and are the standard deviations

$$\rho(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{(X-\mu_X)^2}{2\sigma_x^2} - \frac{(Y-\mu_Y)^2}{2\sigma_Y^2}\right] \qquad (3.2)$$

If the two means are assumed to be equal to zero and the standard deviaton in both the x and y directions are assumed to be equal, then $\rho(x,y)$ can be simplified to the circular normal distribution function of equation 3.3

$$\rho(r) = \frac{1}{2\pi\sigma_r^2} \exp\left(-\frac{r^2}{2\sigma_r^2}\right) \qquad (3.3)$$

The r is the miss distance from the target , and σ_r is the standard deviation of the miss distance. A more descriptive term, the Circle of Equal Probability, or CEP, is often used in place of σ_r. The CEP is the circular miss distance within which one half of the shots fall. The CEP is related to σ_r by equation 3.4.

$$\text{CEP} = 1.177\,\sigma_r \qquad (3.4)$$

C. FUZING PROBABILITY

The fuzing probability, P_f , is an important factor in the P_{KSS} equation. There are many factors that can effect the fuzing probability, including miss distance, relative velocities, and encounter geometry. It can best be thought of as the probability that the warhead fuzes at the proper time to maximize the damage to the target. For a lethality

estimate it is normally considered to be a constant less than one. Sometimes a fuze cutoff range r_c is also used such that P_f equals zero for miss distances greater than r_c. When P_f is considered to be a constant it also becomes a limiting value for the P_{KSS} equation. For example, if $P_f = 0.8$ the maximum obtainable P_{KSS} is also 0.8.

D. KILL FUNCTION APPROXIMATIONS

For proximity fuzed warheads, the kill function $V(x,y)$ or $V(r)$ can be expressed as $P_{K/D}$. Recall that the $P_{K/D}$ function has the form,

$$1 - \exp(\text{constant}/ R^2_{DET}) \qquad (3.5)$$

where R_{DET} is the detonation distance. Combining this expression with the bivariate or circular normal miss distance distribution function results in an expression for P_{KSS} for which there is no closed form solution. This problem can be solved by making an approximation to the P function which, when combined with the miss distance distribution, does lead to a closed form solution. There are two approximations to the $P_{K/D}$ function that are often used. They are the "cookie-cutter" kill function and the Carlton kill function.

1. The Cookie-Cutter Kill Function

Perhaps the simplest kill function is the "cookie-cutter" kill function. Recall that the lethal radius, r_ℓ, is defined as the point where $P_{K/D}$ is equal to 0.5. Usually the lethal radius is used as a cutoff point such that;

$$P_{K/D} = 1 \text{ for } 0 < r < r_\ell$$
$$P_{K/D} = 0 \text{ for } r > r_\ell$$

Figure 3.1 shows the cookie-cutter approximation to the $P_{K/D}$ function.

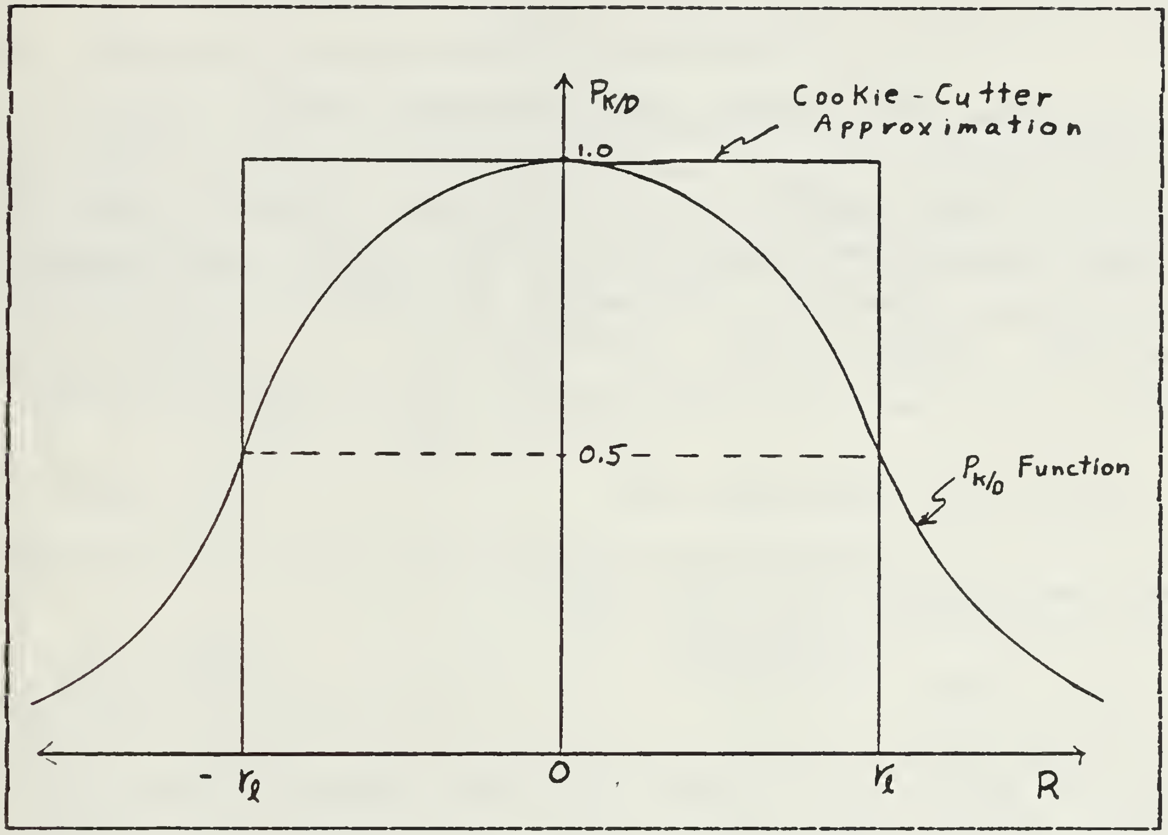


Figure 3.1 Cookie-Cutter Kill Function

For a circular cookie-cutter kill function and a circular normal miss distance distribution the P_{KSS} equation becomes

$$P_{KSS} = \int_0^{2\pi}\int_0^{r_\ell} \frac{P_t}{2\pi\sigma_r^2} \exp\left[-\frac{r^2}{2\sigma_r^2}\right] r\, dr\, d\theta \qquad (3.6)$$

For a square box cookie-cutter kill function and a bivariate normal distribution the P equation becomes

$$P_{KSS} = \int\int \frac{P_t}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right] dx\, dy \qquad (3.7)$$

2. The Carlton Kill Function Approximation

A more accurate method of approximating the kill function is obtained by using the circular Carlton kill function shown in equation 3.8 below

$$V(r) = P_{K/D} = \exp(-r^2/r_o^2) = \exp(-x^2/r_o^2)\exp(-y^2/r_o^2) \qquad (3.8)$$

In the Carlton function, r_o, is a scaling parameter used to adjust the Carlton function to provide a close approximation to the $P_{K/D}$ function. One method of picking r_o is to colocate the Carlton and $P_{K/D}$ functions at $r = 0$ and $r = r_\ell$. When $r = r_\ell$, $P_{K/D} = 0.5$ and equating the two functions gives

$$P_{K/D} = 0.5 = \exp(-r_\ell^2 / r_o^2) \qquad (3.9)$$

solving equation 3.9 for r_o gives

$$r_o = 1.2\, r_\ell \qquad (3.10)$$

Another method is to equate the area under the $P_{K/D}$ function, known as the lethal area, A_L, to the area under the circular kill function, $2\pi r_o^2$, giving

$$r_o = \sqrt{\frac{A_L}{2\pi}} \qquad (3.11)$$

The circular Carlton kill function along with the circular normal miss distance distribution result in the following form of the P equation.

$$P_{KSS} = \int_0^{2\pi}\int_0^{\infty} \frac{P_\ell}{2\pi\sigma_r} \exp\left[\frac{-r^2}{2\sigma_r^2}\right] \exp\left[\frac{-r^2}{r_o^2}\right] r\, dr\, d\theta \qquad (3.12)$$

Using the bivariate normal distribution and the two dimensional Carlton function results in the form of equation 3.13.

$$P_{KSS} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{P_f}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{(X-\mu_x)^2}{2\sigma_x^2} - \frac{(Y-\mu_y)^2}{}\right]\exp\left(\frac{-X^2}{r_0^2}\right) \exp\left(\frac{-Y^2}{r_0^2}\right) dx\, dy \qquad (3.13)$$

E. EVALUATING SINGLE SHOT KILL PROBABILITY

Closed form solutions to the P_{KSS} integrals are desired in order to simplify the calculation of P_{KSS} in the determination of a weapons systems lethality. Consider the P_{KSS} equation obtained from the circular normal distribution and the Carlton kill function, equation 3.12. It can be further simplified to

$$P_{KSS} = \int_0^{2\pi}\int_0^{\infty} C_1 r \exp(C_2 r^2)\, dr\, d\theta \qquad (3.14)$$

where C_1 and C_2 are the constants shown below

$$C_1 = \frac{P_f}{2\pi\sigma_r^2} \qquad C_2 = \left(-\frac{r_0^2 + 2\sigma_r^2}{2\,\sigma_r^2\, r_0^2}\right)$$

Solving the integral above gives

$$P_{KSS} = \frac{C_1 \pi}{C_2} \exp(C_2 r^2)\Big|_{r=0}^{r=\infty}$$

Since C_2 is a negative constant, this becomes

$$P_{KSS} = \frac{C_1\pi}{C_2}(0 - 1) = -\frac{C_1\pi}{C_2}$$

Substituting in the values of C_1 and C_2 gives the closed form solution

$$P_{KSS} = P_f \frac{r_0^2}{r_0^2 + 2\sigma_r^2} \qquad (3.15)$$

where $r_0 = 1.2\, r_l$ or $r_0^2 = A_L/\pi$, depending on which way was chosen to calculate r_0.

Table 1 provides a summary of the P_{KSS} equations for the different combinations of kill functions and miss distance distributions.

TABLE 1

Single Shot Kill Probability Equations

PROXIMITY FUZED WARHEADS

KILL FUNCTION	P_{KSS} EQUATIONS
COOKIE CUTTER (X,Y)	$\int_{-r_l}^{-r_l}\int_{-r_l}^{-r_l} \frac{P_f}{2\pi\sigma_x\sigma_y}\ EXP\left[-\frac{(x-\mu_x)^2}{2\sigma_x} - \frac{(y-\mu_y)^2}{2\sigma_y}\right] dx\,dy$
(r)	$P_f\left[1 - EXP\left(\frac{-r_l}{2\sigma_r^2}\right)\right]$
CARLTON (X,Y)	$\frac{r_o^2}{(2\sigma_x^2+r_o^2)^{1/2}(2\sigma_y^2+r_o^2)^{1/2}}\ P_f\ EXP\left[-\frac{\mu_x^2}{(2\sigma_x^2+r_o^2)} - \frac{\mu_y^2}{(2\sigma_y^2+r_o^2)}\right]$ $r_o = 1.2\, r_l$ or $r_o = \sqrt{A_L/\pi}$
(r)	$P_f \cdot \frac{r_o^2}{2\sigma_r^2+r_o^2}$ $r_o = 1.2\, r_l$ or $r_o = \sqrt{A_L/\pi}$ FUZE CUTOFF CORRECTION FACTOR $= 1 - EXP\left[-\frac{(2\sigma_r^2+r_o^2)}{2\sigma_r^2 r_o^2}\, r_c^2\right]$

F. CORRECTIONS TO THE P_{KSS} EQUATION

1. The Effects of a Fuze Cutoff

If a fuze on the warhead has a cutoff range, r_c, the P_{KSS} for the circular Carlton kill function is nearly the same, except that the limits of integration in the radial direction no longer extend to infinity but must stop at the fuze cutoff range. Changing the upper limit for r in equation 3.14 to r_c and performing the integration result in the following equations

$$P_{KSS} = \frac{C_1 \pi}{C_2} \exp(C_2 r^2) \Big|_{r=0}^{r=r_c}$$

$$= \frac{C_1 \pi}{C_2} \times (1 - \exp(C_2 r_c^2))$$

Note that this corrected P_{KSS} equation is merely equation 3.15 multiplied by a correction factor given by equation 3.16 below

$$\text{fuze cutoff correction} = 1 - \exp(C_2 r_c^2) \qquad (3.16)$$

This correction factor reduces the P_{KSS} because it is always less than one for any finite value of r_c.

2. Correction to P_{KSS} for Sea Skimming Targets

Low flying or sea skimming targets pose an additional problem in that some missiles will impact the water prior to reaching the proper detonation point. The problem is best seen by looking at it in both the intercept plane of Figure 3.2 and in a plane perpendicular to the intercept plane, Figure 3.3.

For R_{DET} greater than $h = H/\sin\gamma_{CF}$ in the intercept plane, where H is the target altitude, the missile impacts the water before reaching R_{DET}. The presence of the ocean

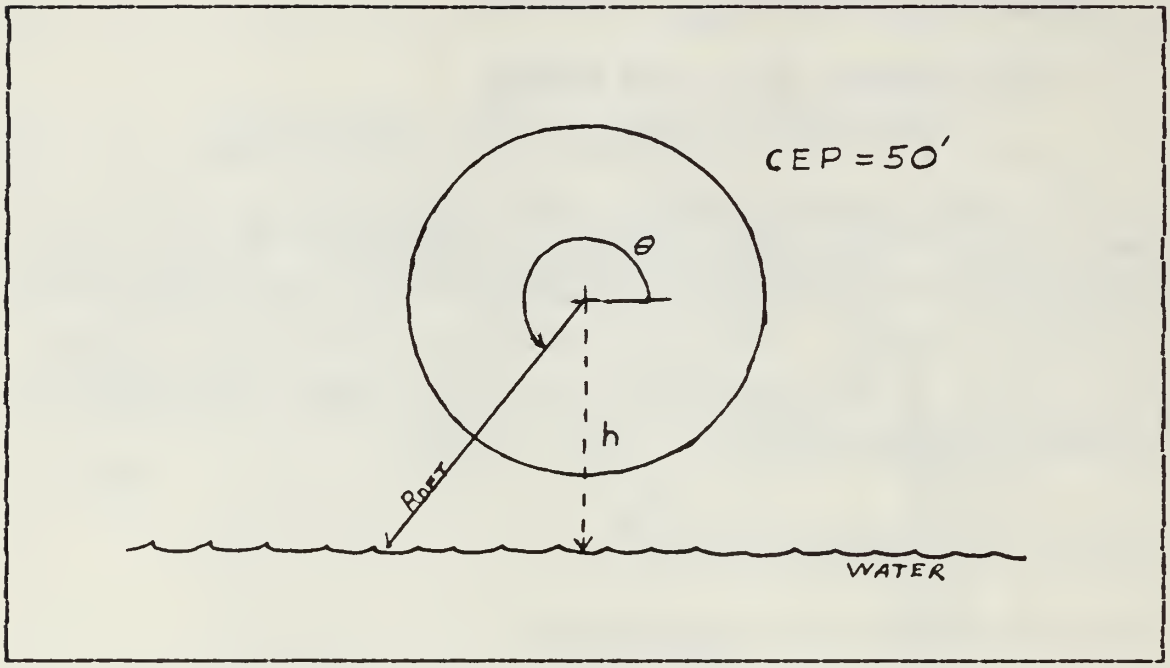


Figure 3.2 Intercept Plane

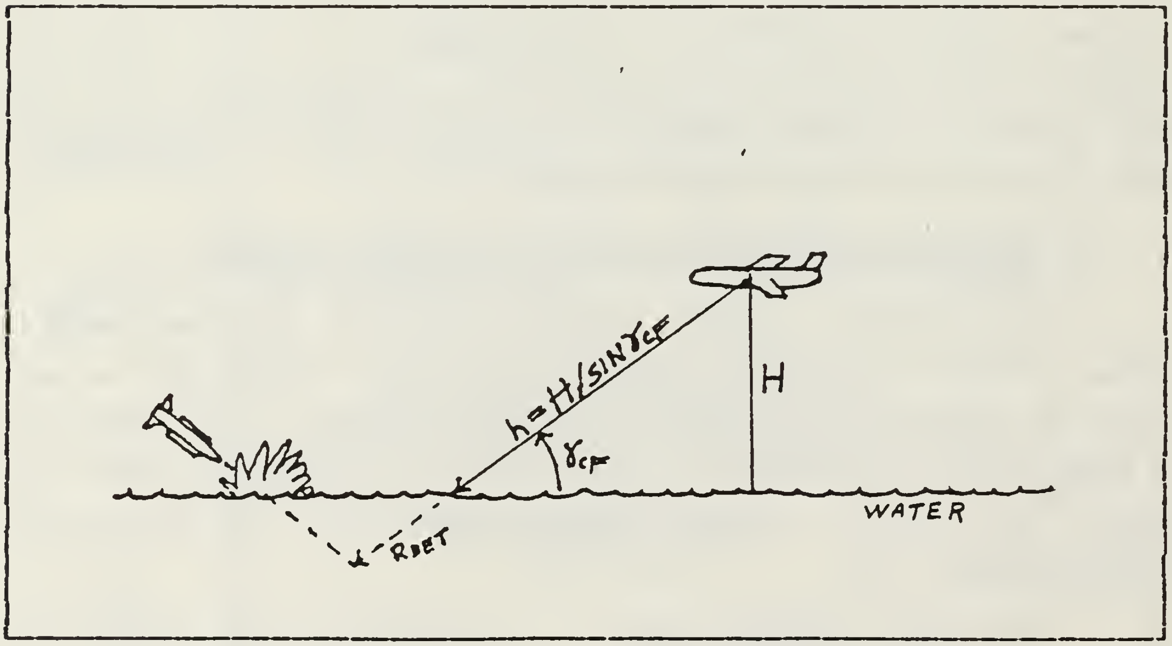


Figure 3.3 Target Plane

affects the limits of integration in the P_{KSS} equation. On the upper half plane r still goes from zero to infinity but on the lower half plane r is limited by the ocean surface to $r = h/\sin(2\pi - \theta)$. The result is to split up the P_{KSS} integral into two integrals, one over the upper half plane, and the other over the lower half plane as follows

$$P_{KSS} = \int_0^{\pi}\int_0^{\infty} C_1 r \exp(C_2 r^2)\, dr\, d\theta$$

$$+ \int_{\pi}^{2\pi}\int_0^{h/\sin(2\pi-\theta)} C_1 r \exp(C_2 r^2)\, dr\, d\theta$$

Solving the first integral we obtain

$$-C_1\pi / 2C_2$$

Reducing the second integral results in

$$-\frac{C_1\pi}{2C} + \frac{C_1}{2C_2}\int_{\pi}^{2\pi} \exp \frac{C_2 h^2}{\sin^2(2\pi-\theta)}\, d\theta$$

Adding the two together and substituting for C and C outside of the integral we obtain

$$P_{KSS} = P_f\left(\frac{r_o^2}{r_o^2 + 2\sigma_r^2}\right) \times \left[1 - \frac{1}{2\pi}\int_{\pi}^{2\pi}\exp\left(\frac{C_2 h^2}{\sin^2(2\pi-\theta)}\right) d\theta\right]$$

Note that this is just the original formula for P_{KSS} multiplied by a correction factor which is less than or equal to one. The integral in the correction factor can be solved using a calculator and Simpson's rule.

G. CUMULATIVE KILL PROBABILITY

The calculation of a single shot kill probability gives a basic measure of a weapon systems lethality. In order to maximize a combat systems lethality, usually more than one weapon is fired at a given target. The cumulative kill probability, P_K, gives a measure of the weapon systems

ability to handle one specific threat. Because P_K is one minus the cumulative probability of survival, P_S, and P_S is the product of the probability of survival for each shot, the equation for P_K is

$$P_K = 1 - P_S = 1 - \prod_{i=1}^{n}(1 - P_{KSS_i}) \qquad (3.17)$$

where n is the total number of shots and P_{KSS_i} is the kill probability of the ith shot. For the case where P_{KSS_i} is a constant for each shot, equation 3.17 reduces to

$$P_K = 1 - (1 - P_{KSS})^n \qquad (3.18)$$

IV. COMBAT SYSTEM LETHALITY

The preceding Chapters developed the methodolgy for determining both a warhead and the weapons systems lethality. Modern combat systems are usually composed of more than one weapons system, along with various sensor systems, which are coordinated by some form of command and control system. The combination of weapons systems, sensor systems, and control systems into a single combat system adds another dimension into the lethality calculations. Time is an extremely important factor in the system lethality. For example, a highly lethal weapon system that takes minutes to react to a threat would not be an effective combat system against multiple targets, even though $P_{KSS} = 1$.

A. MEASURES OF COMBAT SYSTEM LETHALITY

There are two important measures of combat system lethality that must be looked at together in order to adequately assess the lethality of the entire system. The first measure to be considered is the lethality of the weapons sytems which was discussed in the previous chapter. The second measure to be considered is the threat handling capacity of the system in the face of multiple threats.

1. Threat Handling Capacity

A combat system's threat handling capacity is defined to be the maximum number of threats the system can handle without allowing any threat to go unchallenged. It can be considered as the saturation point of the system. Any further increase in the number of threats results in

saturating the combat systems defenses. Any threat that is not engaged by a weapon system, before reaching the weapon system's minimum range, is called a leaker.

2. Threat Survival

Every threat engagement has only two possible outcomes, either the threat is killed or the threat survives. The outcome of a number of engagements is described by the binomial distribution in which the probability of survival, P_S, is one minus the cumulative kill probability, P_K. For a given number of engagements, N_E, the number of survivors, N_S, is estimated using the binomial distribution having the mean, μ_{N_S}, and variance, $\sigma^2_{N_S}$, given by equations 4.1 and 4.2 below

$$\text{average number of survivors} = \mu_{N_S} = N_E P_S \tag{4.1}$$

$$\text{variance} = \sigma^2_{N_S} = N_E P_S P_K \tag{4.2}$$

B. FACTORS AFFECTING COMBAT SYSTEM LETHALITY

A number of factors have an effect on the lethality of a combat system. Some of the more important factors are layered defenses, target detection ranges, system response and engagement times, and the number of simultaneous engagements the system is capable of handling.

1. Target Detection Range

Target detection ranges affect combat system lethality because the time available for the combat system to counter a specific threat increases with target detection range. There are several factors which affect the range at which a target is detected, including radar power levels, target radar cross section, target altitude, and radar

jamming levels. Perhaps the most limiting factor in the case of a low flying cruise missile is the target altitude. Target altitude determines the range to the radar horizon which can be computed using

$$D = 1.23(\sqrt{H} + \sqrt{h}) \qquad (4.3)$$

where D is the distance to the radar horizon in nautical miles, h is the radar height in feet, and H is the target altitude in feet. For low flying targets, the radar horizon is usually much less than the normal target detection range of the radar.

2. System Response, Acquisition, and Engagement Times

A major factor in the lethality of a combat system is the overall time the system takes to respond to a new threat, acquire the threat with a fire control system, and complete an engagement of the threat. The response time is the time required for the system to detect and begin a track of the target. The acquisition time is the time required to designate a track to a fire control radar and have the fire control radar acquire or "lock on" to the target. The engagement time is equal to the aquisition time plus the time of flight of the missile or projectile.

3. Target Separation Interval

Another important factor in threat handling capacity of a combat system is the time interval, Δt, between successive threats. If the time interval between successive threats is less than the engagement time, ET, each successive threat will penetrate to a closer range than the previous threat before being engaged by the system. Given enough threats, the system will eventually be saturated. For intervals greater than the engagement time, the threat handling capacity is limited only by the magazine capacity.

4. Multiple Fire Control Systems

Combat systems with multiple fire control systems have the capability of controlling simultaneous threat engagements. Since each indivivual system can conduct an engagement, the net effect is to increase the interval between targets that each individual system must handle. For example, a combat system with two fire control directors doubles the time interval between threats as seen by an individual fire control system since each director engages only every other target. Equation 4.4 gives the adjusted time interval, Δt^*, as a function of the original time interval and the number of fire control radars available, NFC.

$$\Delta t^* = (\Delta t \;/\; NFC) \tag{4.4}$$

C. DETERMINING COMBAT SYSTEM LETHALITY

In order to determine a combat systems lethality against a specific threat we start by determining the threat handling capacity of the outermost layer of defense, followed by the remaining layers of defense. The following outlines the procedural steps necessary to evaluate threat handling capacity:

(1) determine the target detection range,
(2) determine target range when track is established,
(3) determine range of first intercept,
(4) determine range of subsequent intercepts,
(5) determine the number of threats required to penetrate to the minimum weapon range or next layer of defense (threat handling capacity),
(6) estimate the number and interval of threats the next defense layer will be confronted with,
(7) repeat steps 4, 5, and 6 for each remaining level of defense.

1. Target Detection Range

Target detection range will be the lesser of the following ranges: (1) the minimum expected burn through range for a given target cross section and level of jamming, or (2) the radar horizon range for the given target altitude as given by equation 4.3. This target detection range can be corrected for the additional distance the target will travel during the finite time it takes for the search radar antenna to rotate once.

2. Track Range

Once a target is detected it normally takes at least three data points to establish a track. The range at which a track is established, TR, is the detection range, DR, plus the product of target velocity, V_T, and the time required for at least two more rotations of the radar antenna, ARI.

$$TR = DR + (V_T \times 2 \times ARI) \tag{4.5}$$

3. Intercept Ranges

Assuming the decision to engage the target has been made by the time a track is established, we now want to know at what range the first intercept, IR_1, and subsequent intercepts, IR_n, will occur. The first intercept range can be found by subtracting the distance the target travels during the time of flight, TOF, from the launch range, LR, as shown in equation 4.6.

$$IR_1 = LR_1 + (TOF \times V_T) \tag{4.6}$$

where TOF is equal to the launch range divided by the relative velocity of the two missiles

$$TOF_1 = LR_1 / (V_{M_1} - V_{T_1}) \tag{4.7}$$

The initial launch range is equal to the track range plus the distance the target travels during the aquisition time, AT, and is given by

$$LR_1 = TR_1 + (AT_1 \times V_{T_1}) \tag{4.8}$$

Substituting equations 4.8 and 4.7 into equation 4.6 results in equation 4.9

$$IR_1 = (TR_1 + (AT_1 \times V_{T_1})) \times VR_1 \tag{4.9}$$

where VR_1 is the velocity ratio $(V_{M_1} / (V_{M_1} - V_{T_1}))$.

At the time of the first intercept the second target will be at a range equal to or greater than the first intercept range, depending on the value of Δt^*, as given by

$$\text{target two range} = IR_1 - (\Delta t^* \times V_T) \tag{4.10}$$

The second intercept range can be found by substituting the appropriate subscripts for target two into equation 4.9 giving

$$IR_2 = ((IR_1 - \Delta t^* V_T) + AT_2 V_T) \times VR_2 \tag{4.11}$$

Assuming all parameters remain the same for each engagement, equation 4.11 can be made into a recursion formula to find the subsequent intercept ranges

$$IR_{n+1} = (IR_n + V_T (AT - \Delta t^*)) \times VR \tag{4.12}$$

4. Threat Handling Capacity

Threat handling capacity can now be determined by comparing the subsequent intercept ranges to the minimum range of the weapon system, R_{min}. When the (n+1)th intercept range is less than R_{min} then the system has reached saturation and the threat capacity is equal to n. Thus

$$TC = n \quad \text{for } IR_{n+1} < R_{min} \tag{4.13}$$

5. Subsequent Layers

In order to estimate the threat encountered by the next layer of defense, the number of leakers, N_L, and the number of survivors, N_S, must be calculated for a specified number of attacking threats.

a. The Number of Leakers

Once a threat is determined to be a leaker, ($IR_{n+1} < R_{min}$), subsequent intercept ranges are calculated on the basis that the fire control system skips over the leakers and engages the first target with an intercept range greater than R_{min}. These intercept ranges are given by

$$IR_{L+i} = (IR_L + V_T(AT - i\Delta t^*)) \times VR \tag{4.14}$$

where IR_L is the last valid intercept range obtained, and i is an integer counter which is increased by one until an intercept range is found that is greater than R_{min}. By calculating intercept ranges in this manner, the total number of engagements, N_E, can be determined for a specified number of threats.

b. Number of Survivors

The average number of survivors, N_S, is determined from N_E and the cummulative probability of kill P_K using equation 4.1.

c. Threat Interval For Next Layer

The average threat interval for the next layer of defense can be determined from N_L, N_E, and the threat interval Δt_1 of the previous layer. The average threat interval is computed by taking the total time for a specified number of threats, divided by the number of threats reaching the next layer, as shown below

$$\Delta t_2 = \frac{(N_L + N_E)\Delta t_1}{N_L + (1-P_K)N_E} \qquad (4.15)$$

Prior to saturation N is zero and the average threat interval for the next layer reduces to

$$\Delta t_2 = \frac{N_E \Delta t_1}{(1-P_K)N_E} = \frac{N_E \Delta t_1}{N_S} \qquad (4.16)$$

Calculating Δt_2 prior to level one saturation gives a measure of how well the second level compliments the first. Calculating Δt_2 after level one is saturated is useful to determine if level two will also be saturated. Figure 4.1 is a diagram showing intercept ranges, threat saturation, and threat intervals.

6. <u>Overall Threat Capacity</u>

The threat handling capacity for the entire combat system is the sum of the individual threat capacities for each layer of defense.

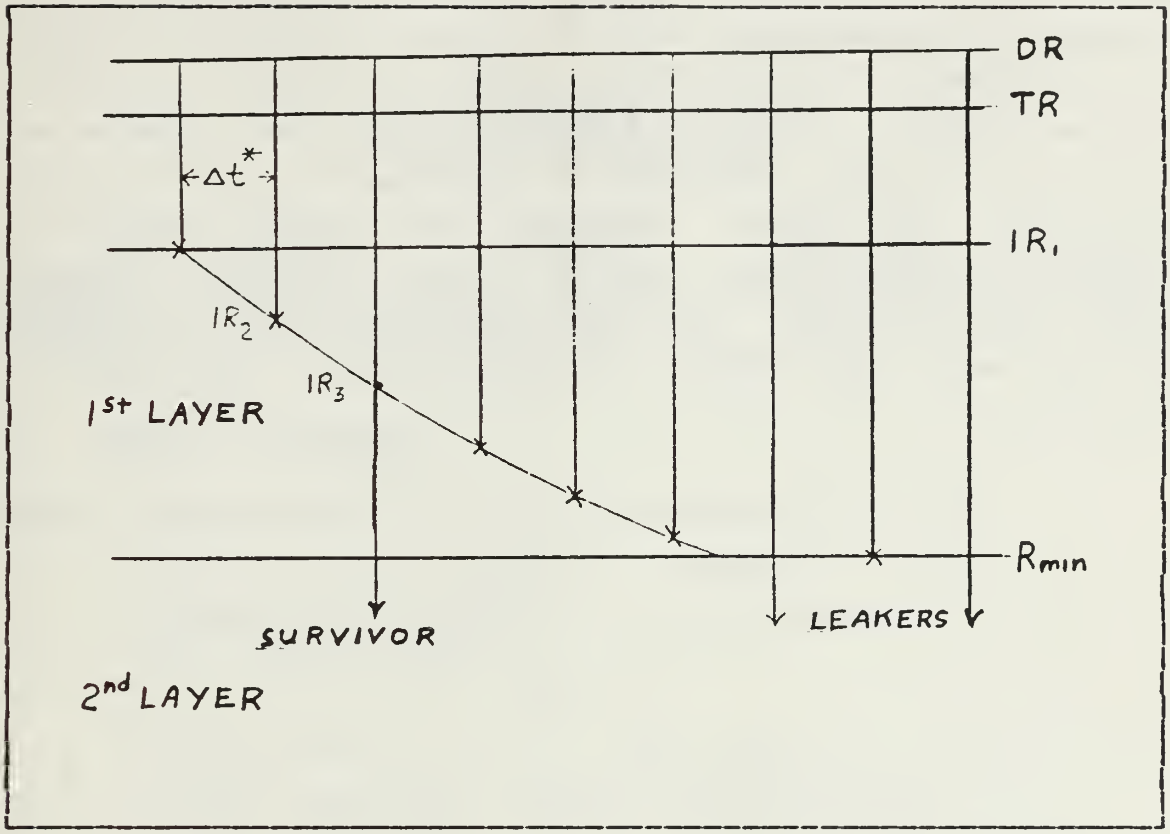


Figure 4.1 Combat System Saturation

D. SUMMARY AND CONCLUSIONS

This thesis develops a methodology for determining the effectiveness of a combat system. First, the factors affecting warhead lethality were analyzed. The warhead was then integrated into a weapon system and the factors affecting the weapon system were analyzed. Finally the weapon system was integrated into a combat system and the factors affecting combat system lethality were analyzed. A template was developed to guide students in the problem solution and two case studies were provided as examples.

The methodology developed in this thesis incorporates a number of simplifying assumptions to allow a student to perform these calculations by hand or with the aid of a

calculator. The potential exists for this methodology to be expanded in key areas and then to develop a computer program to accomplish the calculations. With the aid of a computer program students should be able to analyze the effects of changes in specific parameters on the overall combat system lethality. For example, students could examine changes in warhead characteristics, weapon system accuracy, combat system response times, or target characteristics. Studies of this nature may lead to improvements in combat system lethality.

APPENDIX A
LETHALITY TEMPLATE

This template is to be used as a guide for calculating weapon system lethality in the form of a single shot kill probability and a cumulative kill probability.

A. WARHEAD PARAMETERS

GIVEN: charge weight C = ______ or density = ______

metal weight M = ______ or density = ______

Guerney constant $\sqrt{2E}$ = _____ or explosive: ______

static spray angles α_1 = ____

α_2 = ____

fragment size = ______ and density = ______

warhead geometry: (drawing or dimensions)

FIND: fragment weight in grains = _______

charge to mass ratio C/M = _______

fragment initial velocity V_0 = _______

total number of fragments N = _______

(N may be adjusted to reflect percentage of case that fragments)

Useful Equations:

1 pound = 7000 grains

Guerney formula $$V_0 = \sqrt{2E}\sqrt{\frac{C/M}{1+C/2M}}$$

B. TARGET DESCRIPTION

GIVEN: target velocity V_T = ______

target drawing: including dimensions, component locations, and sizes.

DETERMINE:

Presented area A_p = ______

Minimum kill categories required:

(function of range and velocity)

Critical components for selected kill categories:

ESTIMATE: component $P_{k/h}$ for the desired kill categories:

CALCULATE: Vulnerable area from one or more aspects.

EQUATIONS:

component vulnerable area;

$$A_v = A_p \times P_{k/h}$$

total vulnerable area;

$$A_V = \sum A_v$$

C. ENDGAME ANALYSIS

GIVEN:

WARHEAD PARAMETERS:

Early or late bird = ______ ______

static warhead α_1 = ______ ______

spray angles α_2 = ______ ______

fragment number N = ______

fragment velocity V_0^* = ______

ENCOUNTER PARAMETERS:

Missile/Projectile

velocity V_M or V_P = ______

missile approach angle θ_M = ______

target velocity V_T = ______

target vulnerable areas A_V

top/side = ______ ______

front/rear = ______ ______

(* values may be adjusted to reflect average velocities)

FIND:

dynamic spray angles γ_1 = ______ ______

γ_2 = ______ ______

ϕ_1 = ______ ______

ϕ_2 = ______ ______

γ_{CF} = ______ ______

range ratio R/R_{DET} = ______ ______

fragment spray densities

as a function of R $\rho(R)$ = ______ ______

as a function of R_{DET} $\rho(R_{DET})$ = ______ ______

projected vulnerable area

projected $A_V(\gamma_{CF})$ = ______ ______

Probability of a kill given a

detonation $P_{K/D}(\rho(R_{DET}), A_V(\gamma_{CF}))$ = ______ ______

(calculate at various points as desired)

EQUATIONS:

$$\gamma_i = \arctan\left(\frac{V_M \sin(\theta) + V_o \sin(\theta + \alpha_i)}{V_M \cos(\theta) + V_o \cos(\theta + \alpha_i) - (V_T)}\right)$$

$$\phi_i = \gamma_i - \theta$$

$$\gamma_{cf} = (\gamma_1 + \gamma_2)/2$$

$$\frac{R}{R_{DET}} = \frac{|\vec{V}_{fa}|}{|\vec{V}_{f/T}|}$$

$$|\vec{V}_{fa}| = \sqrt{(V_M + V_o \cos\alpha_{cF})^2 + (V_o \sin\alpha_{cF})^2}$$

$$|\vec{V}_{f/T}| = \sqrt{(V_{fa} \cos(\gamma_{cF}) - V_T)^2 + (V_{fa} \sin\gamma_{cF})^2}$$

$$\rho(R) = \frac{N}{2\pi \times R^2 \times (\cos(\phi_1) - \cos(\phi_2))}$$

$$\text{Projected } A_V = A_{V_1} |\sin(\gamma_{cF})| + A_{V_2} |\cos(\gamma_{cF})|$$

$$P_{K/D} = 1 - \exp(-\rho \times \text{projected } A_V)$$

D. DETERMINATION OF WEAPON SYSTEM LETHALITY

GIVEN:

Circle of Equal Probability CEP = ______

or

standard deviation σ_r = ______

fuzing probability P_f = ______

fuze cutoff range r_c = ______

fragment spray density as a function of R_{DET} $\rho(R_{DET})$ = ______

projected vulnerable area $A_V(\gamma_{cF})$ for desired kill levels $A_V(\gamma_{cF})$ = ______KK

= ______K

= ______A

= ______B

target altitude H = ______

center spray angle γ_{cF} = ______

Firing doctrine:

FIND:

lethal radius (r_ℓ) for desired kill levels r_ℓ = ______KK

= ______K

= ______A

= ______B

Carlton scaling parameter (r_o) for desired kill levels r_o = ______KK

= ______K

= ______A

= ______B

single shot kill probability (P_{KSS}) for desired kill levels P_{KSS} = ______KK

= ______K

= ______A

= ______B

fuze cutoff correction factor = ______

sea skimmer correction factor = ______

corrected kill probability P_{KSS} = ______

cumulative kill probability
(weapon system lethality) P_K = ______

EQUATIONS:

$$CEP = 1.177\,\sigma_r$$

$$P_{K/D} = 0.5 = 1 - \exp(-\rho(r_o)\,A_V)$$

$$P_{KSS} = P_f \,\frac{r_o^2}{r_o^2 + 2(\sigma_r)^2}$$

$$\text{fuze cutoff factor} = (1 - \exp(C_2\, r_c^2)$$

$$C_2 = \left(- \frac{r_o^2 + 2\sigma_r^2}{2\,\sigma_r^2\, r_o^2}\right)$$

sea skimmer correction factor;

$$1 - \frac{1}{2\pi}\int_{\pi}^{2\pi} \exp\left[\frac{C_2\, h^2}{\sin^2(2\pi - \theta)}\right] d\theta$$

$$h = H / \sin(\gamma_{CF})$$

$$P = 1 - \prod_{i=1}^{n} (1 - P_{KSS_i})$$

E. DETERMINATION OF COMBAT SYSTEM LETHALITY

GIVEN:

TARGET DATA:

Cumulative kill probability	P_K =	______
Target altitude	H =	______
Target velocity	V_T =	______
Target separation interval	Δt =	______
Number of attacking threats	N_T =	______

COMBAT SYSTEM DATA:

Missile/projectile velocity	V_M =	______
fire control acquisition time	AT =	______
number of fire control radars	NFC =	______
search radar height	h =	______
antenna rotation interval	ARI =	______
weapon system minimum range	R_{min} =	______

FIND:

target detection range	DR =	______
track range	TR =	______
velocity ratio	VR =	______
range of first intercept	IR_1 =	______
range of successive intercepts	IR_2 =	______
	IR_3 =	______

$IR_4 =$ ______

$IR_5 =$ ______

$IR_6 =$ ______

$IR_7 =$ ______

$IR_8 =$ ______

$IR_9 =$ ______

$IR_{10} =$ ______

$IR_{11} =$ ______

threat handling capacity $TC =$ ______

average target separation interval
for the next defense layer

before saturation $\Delta t_2 =$ ______

after saturation $\Delta t_2 =$ ______

EQUATIONS:

radar horizon $D = 1.23 \ (\sqrt{H} + \sqrt{h})$

$\{D\} =$ nm. $\{H\}$ and $\{h\} =$ ft.

$DR = D + (0.5 \times ARI \times V_T)$

$TR = DR + (V_T \times 2\ ARI)$

$VR = V_M \ / \ (V_M - V_T)$

$IR_1 = (TR + (AT \times V_T)) \times VR$

$IR_{n+1} = \{ IR_n + V_T (AT - \Delta t^*) \} \times VR$

$\Delta t_1^* = \Delta t_1 \ / \ NFC$

$TC = n$ for $IR_{n+1} < R_{min}$

$IR_{L+i} = (IR_L + V_T (AT - i\Delta t^*)) \times VR$

$N_S = N_E \ (1 - P_K)$

$\Delta t_2 = (N_L + N_E) \Delta t_1 \ / \ (N_L + N_S)$

APPENDIX B

SAM SYSTEM VS. CRUISE MISSILE

This is a case study of a generic surface to air missile (SAM) system versus a generic sea skimming cruise missile. The object of the case study is to determine the lethality of the weapon system against the cruise missile and also to assess the contribution of the weapon system to the overall lethality of the associated combat system.

A. WARHEAD PARAMETERS

GIVEN: charge weight C = ___—___ or density = .0596 lbs./in^3

metal weight M = ___—___ or density = .283 lbs/in^3

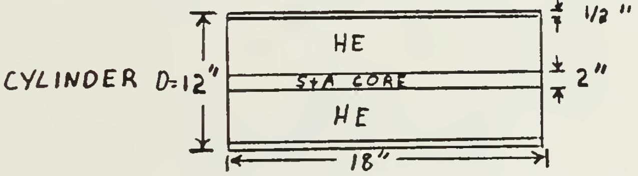
Guerney constant $\sqrt{2E}$ = 9300 or explosive: RDX

static spray angles α_1 = 80

α_2 = 110

fragment size = $.5^3$ in^3 and density = .283 lbs/in^3

warhead geometry: (drawing or dimensions)



FIND: fragment weight in grains = 248

charge to mass ratio C/M = 1.07

fragment initial velocity V_0 = 7765

total number of fragments N = 2601

(N may be adjusted to reflect percentage of case that fragments)

Useful Equations:

1 pound = 7000 grains

Guerney formula $$V_0 = \sqrt{2E}\sqrt{\frac{C/M}{1+C/2M}}$$

CALCULATIONS:

charge weight: C

$18in \times (\pi/4)(11^2 - 2^2)in^2 \times .0596\ lb/in^3 = 98.6\ lbs.$

case weight: M

$18in \times (\pi/4)(12^2 - 11^2)in^2 \times .283\ lb/in^3 = 92\ lbs.$

fragment weight:

$(.5^3)in^3 \times .283\ lb/in^3 \times 7000\ gr/lb = 247.6\ gr.$

charge to mass ratio: C/M = 98.6 / 92 = 1.07

fragment initial velocity:

$V_o = 9300 \times \sqrt{1.07/(1 + (1.07/2))} = 7765\ fps$

number of fragments: N

$(92lbs \times 7000\ gr/lb)/\ 247.6\ gr/fragment = 2601$

B. TARGET DESCRIPTION

GIVEN: target velocity V_T = -1000 fps

target drawing: including dimensions, component locations, and sizes.

see figure B.1

DETERMINE:

Presented area A_p = 28.5 $ft.^2$

Minimum kill categories required:

(function of range and velocity)

K > 10,000 yds. (30 sec. from impact)

KK < 10,000 yds.

Critical components for selected kill categories:

guidance, warhead, fuel tank, engine

ESTIMATE: component $P_{k/h}$ for the desired kill categories:

see table 2

CALCULATE: Vulnerable area from one or more aspects.

see table 2

EQUATIONS:

component vulnerable area;

$$A_v = A_p \times P_{k/h}$$

total vulnerable area;

$$A_V = \sum A_v$$

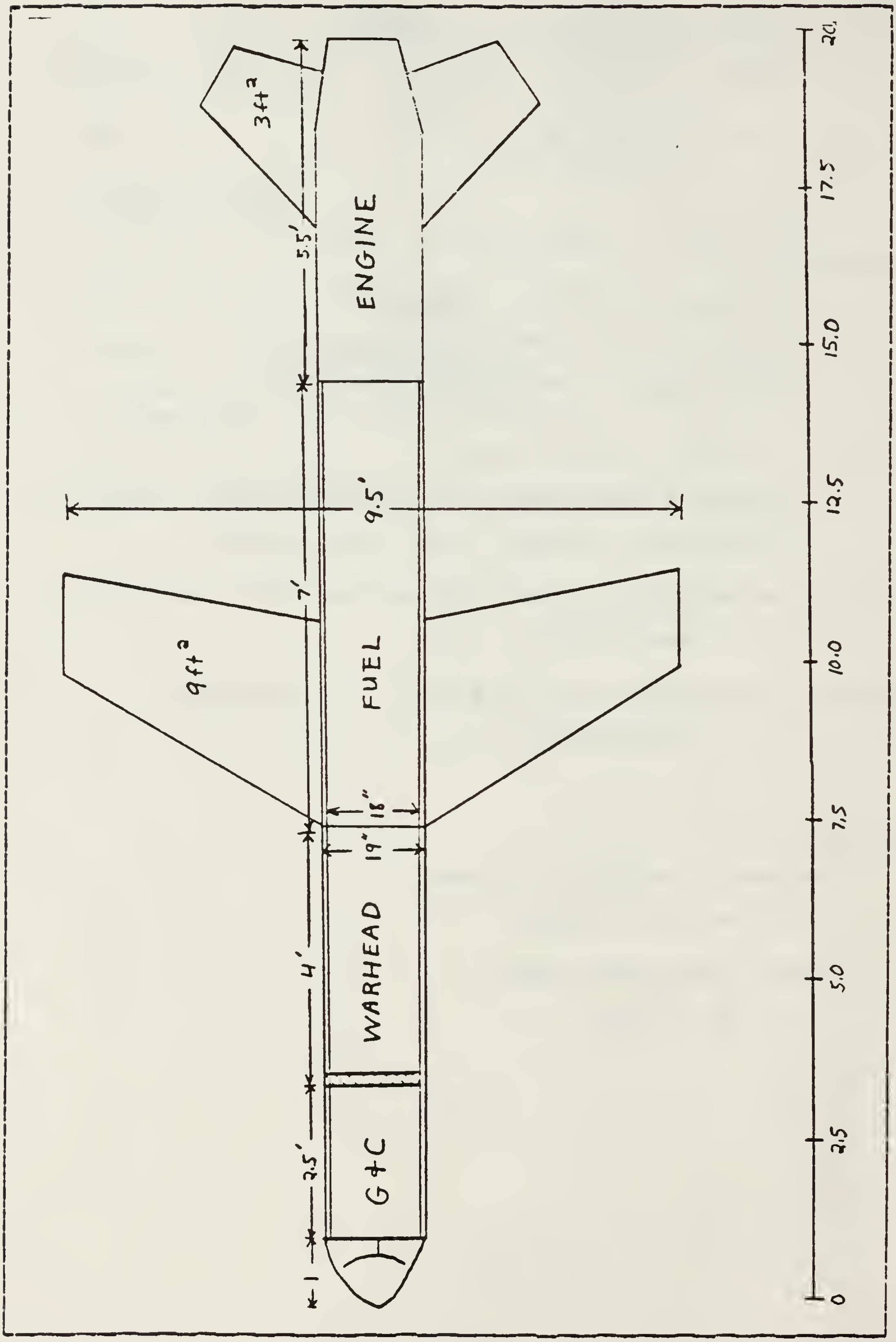


Figure B.1 Cruise Missile Cross-section

TABLE 2

Sea Skimming Cruise Missile Vulnerable Area

SIDE VIEW

Critical Components	A_p	$P_{k/h}$(K)	$P_{k/h}$(KK)	A_v(K)	A_v(KK)
guidance and control	3.75	.3	.1	1.125	.375
warhead	6.00	.1	.1	.6	.6
fuel tank	10.5	.2	.1	2.10	1.05
engine	8.25	.2	.1	1.65	.825
TOTAL	28.5ft^2			5.475ft^2	2.85ft^2

FRONT VIEW

Critical Components	A_p	$P_{k/h}$(K)	$P_{k/h}$(KK)	A_v(K)	A_v(KK)
guidance and control	1.77	.3	.1	.53	.18

TAIL VIEW

Critical Components	A_p	$P_{k/h}$(K)	$P_{k/h}$(KK)	A_v(K)	A_v(KK)
engine	1.77	.2	.1	.35	.18

C. ENDGAME ANALYSIS

GIVEN:

WARHEAD PARAMETERS:

Early or late bird	=	LATE	EARLY
static warhead	α_1 =	-80	80
spray angles	α_2 =	-110	110
fragment number	N =	2600	
fragment velocity	V_0^* =	7000	

ENCOUNTER PARAMETERS:

Missile/Projectile velocity	V_M or V_P =	2000	
missile approach angle	θ_M =	$-30°$	
target velocity	V_T =	-1000 fps	
target vulnerable areas A_V	top/side =	5.475	5.475
	front/rear =	.53	.35

(* values may be adjusted to reflect average velocities)

FIND:

dynamic spray angles	γ_1 =	-87	31
	γ_2 =	-115.6	56.2
	ϕ_1 =	-57	61
	ϕ_2 =	-85.6	86.2
	γ_{cF} =	-101.3	43.6
range ratio	R/R_{DET} =	1.02	.90
fragment spray densities as a function of R	$\rho(R)$ =	$884/R^2$	$988/R^2$
as a function of R_{DET}	$\rho(R_{DET})$ =	$772/R_{DET}^2$	$1220/R_{DET}^2$
projected vulnerable area	projected $A_V(\gamma_{cF})$ =	5.435	4.16
Probability of a kill given a detonation	$P_{K/D}(\rho(R_{DET}), A_V(\gamma_{cF}))$ =	.92 (40.4')	.96 (40.4')

(calculate at various points as desired)

EQUATIONS:

$$\gamma_i = \arctan\left(\frac{V_M \sin(\theta) + V_o \sin(\theta + \alpha_i)}{V_M \cos(\theta) + V_o \cos(\theta + \alpha_i) - (V_T)}\right)$$

$$\phi_i = \gamma_i - \theta$$

$$\gamma_{CF} = (\gamma_1 + \gamma_2)/2$$

$$\frac{R}{R_{DET}} = \frac{|\vec{V}_{fa}|}{|\vec{V}_{f/T}|}$$

$$|\vec{V}_{fa}| = \sqrt{(V_M + V_o \cos\alpha_{CF})^2 + (V_o \sin\alpha_{CF})^2}$$

$$|\vec{V}_{f/T}| = \sqrt{(V_{fa} \cos(\gamma_{CF}) - V_T)^2 + (V_{fa} \sin\gamma_{CF})^2}$$

$$\rho(R) = \frac{N}{2\pi \times R^2 \times (\cos(\phi_1) - \cos(\phi_2))}$$

$$\text{Projected } A_V = A_{V_1} |\sin(\gamma_{CF})| + A_{V_2} |\cos(\gamma_{CF})|$$

$$P_{K/D} = 1 - \exp(-\rho \times \text{projected } A_V)$$

CALCULATIONS:

LATE BIRD

Dynamic spray angles:

$$\gamma_1 = \arctan\left(\frac{2000\ \sin(-30) + 7000\ \sin(-30\ -80)}{2000\ \cos(-30) + 7000\ \cos(-30\ -80) - (-1000)}\right)$$

$$= -87$$

$$\gamma_2 = \arctan\left(\frac{2000\ \sin(-30) + 7000\ \sin(-30\ -110)}{2000\ \cos(-30) + 7000\ \cos(-30\ -110) - (-1000)}\right)$$

$$= -115.6$$

$$\gamma_{CF} = (-115.6 - 87)/2 = -101.3$$

$$\phi_1 = -87 - (-30) = -57$$

$$\phi_2 = -115.6 - (-30) = -85.6$$

$$\alpha_{CF} = (-80 - 110)/2 = -95$$

R/R_{DET} :

$$|V_{fa}| = \sqrt{(2000 + 7000\ \cos(-95))^2 + (7000\ \sin(-95))^2} = 7110$$

$$|V_{f/T}| = \sqrt{(7110\ \cos(-101.3) - (-1000))^2 + (7110\ \sin(-101.3))^2}$$

$$= 6983$$

$$R/R_{DET} = 7110/6983 = 1.07$$

$$\rho(R) = \frac{2600}{2\ R^2\ (\cos(-57) - \cos(-85.6))} = \frac{884}{R^2}$$

$$\rho(R_{DET}) = \frac{884}{R^2_{DET}\ (1.07)^2} = \frac{772}{R^2_{DET}}$$

Projected A_V:

$$A_V(\gamma_{CF}) = 5.475\,|\sin(-101.3)| + .35\,|\cos(-101.3)| = 5.435$$

$P_{K/D}$:

First find where fragment spray covers the target.
Knowing two angles, $\gamma_1 = 87$ and $(\gamma_2 - \gamma_1)/2 = 14.3$
and one side, side = target length / 2 = 10

$R_{DET} = 40.4$ ft. to cover target

$P_{K/D}$ at 40.4 ft. $= 1 - \exp((-772/40.4^2) \times 5.435) = .92$

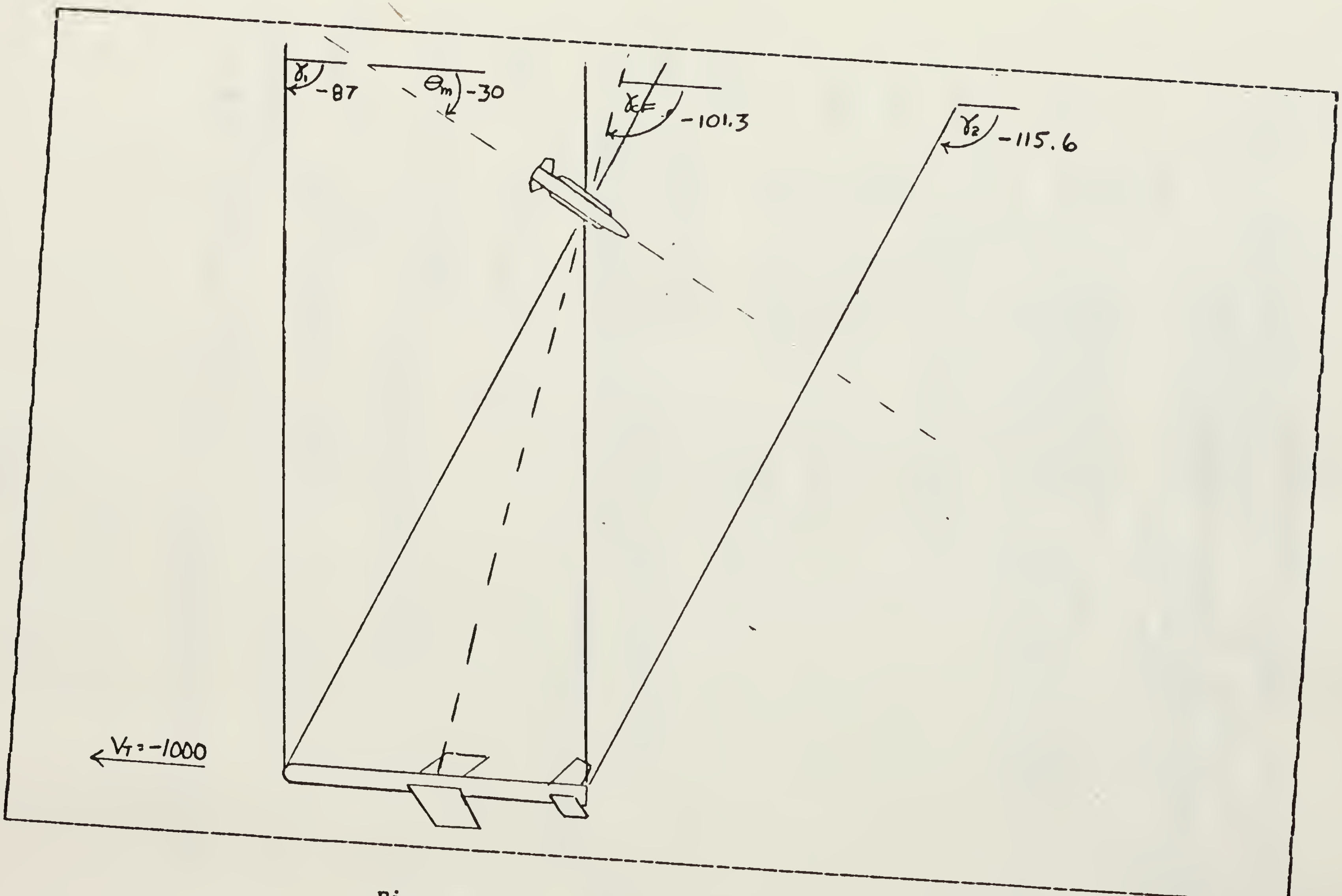


Figure B.2 Late Bird Spray Zones

Dynamic spray angles:

$$\gamma_1 = \arctan\left(\frac{2000\ \sin(-30) + 7000\ \sin(-30 + 80)}{2000\ \cos(-30) + 7000\ \cos(-30 + 80) - (-1000)}\right)$$

$$= +31$$

$$\gamma_2 = \arctan\left(\frac{2000\ \sin(-30) + 7000\ \sin(-30 + 110)}{2000\ \cos(-30) + 7000\ \cos(-30 + 110) - (-1000)}\right)$$

$$= + 56.2$$

$$\gamma_{CF} = (31 + 56.2) / 2 = 43.6$$

$$\phi_1 = 31 - (-30) = 61$$

$$\phi_2 = 56.2 - (-30) = 86.2$$

$$\alpha_{CF} = (80 + 110)/2 = +95$$

R/R_{DET} :

$$|\vec{V}_{fa}| = \sqrt{(2000 + 7000\ \cos(95))^2 + (7000\ \sin(95))^2} = 7110$$

$$|\vec{V}_{f/T}| = \sqrt{(7110\ \cos(43.6) - (-1000))^2 + (7110\ \sin(43.6))^2}$$

$$= 7864$$

$$R/R_{DET} = 7110/7864 = .90$$

$$\rho(R) = \frac{2600}{2\ R^2\ (\cos(61) - \cos(86.2))} = \frac{988}{R^2}$$

$$\rho(R_{DET}) = \frac{988}{R_{DET}^2\ (0.90)^2} = \frac{1220}{R_{DET}^2}$$

Projected A_V:

$$A_V(\gamma_{CF}) = 5.475|\sin(43.6)| + .53|\cos(43.6)| = 4.16$$

$P_{K/D}$:

First find where fragment spray covers the target. Knowing two angles, $\gamma_2 = 56.2$ and $(\gamma_2 - \gamma_1)/2 = 12.6$ and one side, side = target length / 2 = 10

R_{DET} = 23.6 ft. to cover target

$P_{K/D}$ at 23.6 ft. = $1 - \exp((-1220/23.6^2) \times 4.16) = .9999$

$P_{K/D}$ at 40.4 ft. = $1 - \exp((-1220/40.4^2) \times 4.16) = .96$

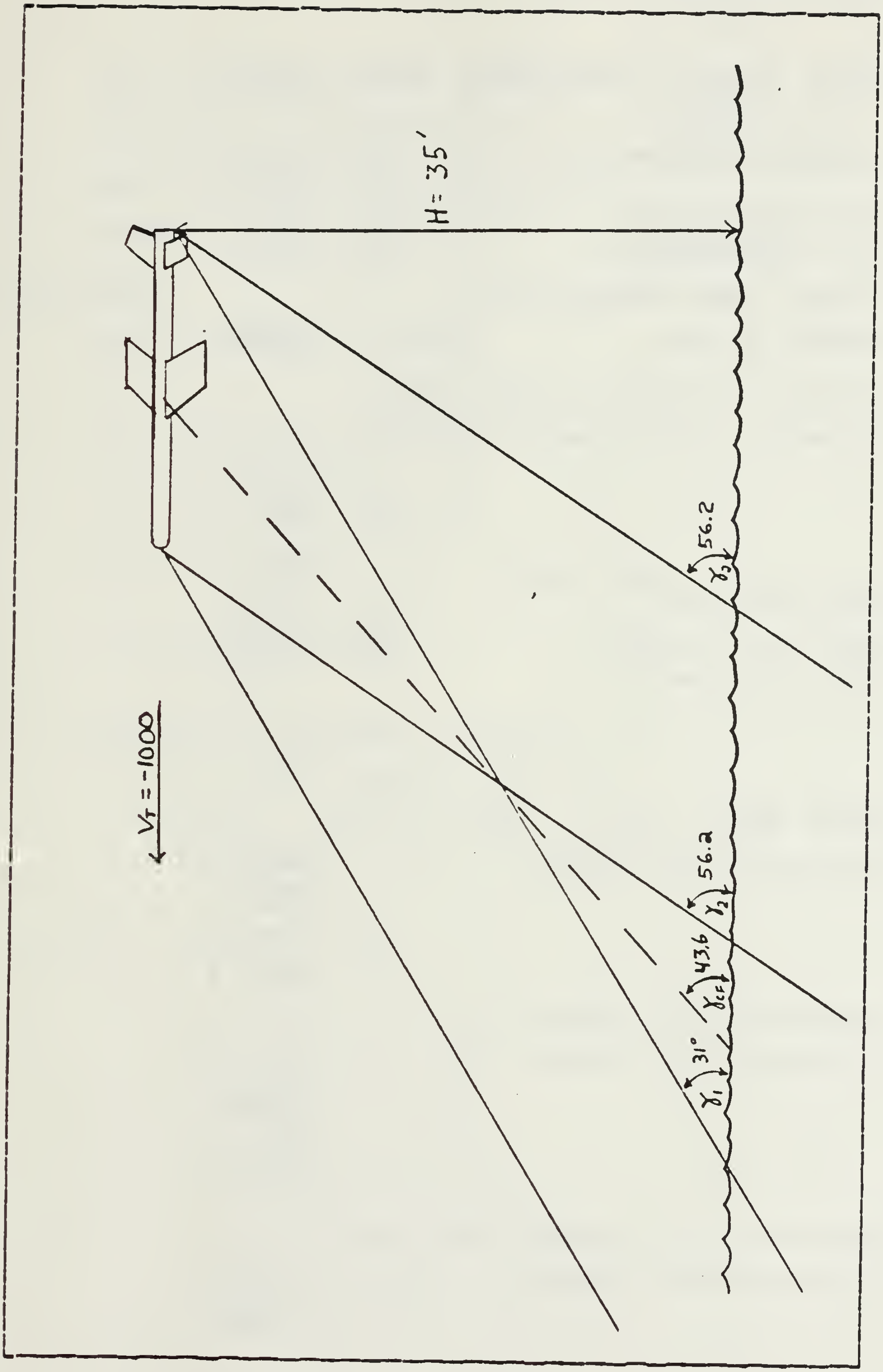


Figure B.3 Early Bird Spray Zones

D. DETERMINATION OF WEAPON SYSTEM LETHALITY

GIVEN:

Circle of Equal Probability	CEP =	50 ft.
or		
standard deviation	σ_r =	42.5 ft.
fuzing probability	P_f =	0.8
fuze cutoff range	r_c =	NA
fragment spray density as a function of R_{DET}	$\rho(R_{DET})$ =	$1220/R_{DET}^2$
projected vulnerable area $A_V(\gamma_{CF})$ for desired kill levels	$A_V(\gamma_{CF})$ =	2.10 KK
	=	4.16 K
	=	NA A
	=	NA B
target altitude	H =	35 ft.
center spray angle	γ_{CF} =	43.6
Firing doctrine:		

FIND:

lethal radius (r_ℓ) for desired kill levels	r_ℓ =	60.9′ KK
	=	85.7′ K
	=	NA A
	=	NA B
Carlton scaling parameter (r_o) for desired kill levels	r_o =	73′ KK
	=	102.8′ K
	=	NA A
	=	NA B
single shot kill probability (P_{KSS}) for desired kill levels	P_{KSS} =	.48 KK
	=	.60 K

= NA A

= NA B

fuze cutoff correction factor		= NA
sea skimmer correction factor		= .91K | .94KK
corrected kill probability	P_{KSS}	= .54K | .45KK
cumulative kill probability (weapon system lethality)	P_K	= .79K | .70KK

EQUATIONS:

$$CEP = 1.177\,\sigma_r$$

$$P_{K/D} = 0.5 = 1 - \exp(-\rho(r_o)\,A_V)$$

$$P_{KSS} = P_f\left(\frac{r_o^2}{r_o^2 + 2(\sigma_r)^2}\right)$$

$$\text{fuze cutoff factor} = (1 - \exp(C_2\, r_o^2)$$

$$C = \left(-\frac{r_o^2 + 2\sigma_r^2}{2\,\sigma_r^2\, r_o^2}\right)$$

sea skimmer correction factor;

$$1 - \frac{1}{2\pi}\int_{\pi}^{2\pi} \exp\left(\frac{C_2 h^2}{\sin^2(2\pi - \theta)}\right) d\theta$$

$$h = H/\sin(\gamma_{cF})$$

$$P_K = 1 - \prod_{i=1}^{n}(1 - P_{KSS_i})$$

CALCULATIONS:

Note: Data for $\rho(R_{DET})$, A_V, and γ_{CF} are from the early bird calculations.

Lethal radius: ($r_\ell = R_{DET}$ for $P_{K/D} = 0.5$)

$$.5 = 1 - \exp((-1220/R_{DET}^2)A_V)$$

$$\ln .5 = (-1220/R_{DET}^2)A_V$$

		K	KK
$r_\ell = 42\sqrt{A_V}$	=	85.7	60.9
$r_o = 1.2\ r_\ell$	=	102.8	73.0
$P_{KSS} = .8\left(\dfrac{r_o^2}{r_o^2 + 2(42.5)^2}\right)$	=	.60	.48

Sea skimmer correction;

$$h = 35/\sin(43.6) = 51'$$

$C_2 = -\dfrac{r_o^2 + 2(42.5)^2}{2(42.5)^2\ r_o^2}$	=	-3.9E-4	-5E-4
$1 - \dfrac{1}{2\pi}\displaystyle\int_{\pi}^{2\pi} \exp\left(\dfrac{C_2 h^2}{\sin^2(2\pi - \theta)}\right) d\theta$	=	.91	.94
corrected P_{KSS}	=	.54	.45

cummulative P_K: (2 bird salvo)

$P_K = 1 - (1 - P_{KSS})^2$	=	.79	.70

E. DETERMINATION OF COMBAT SYSTEM LETHALITY

GIVEN:

TARGET DATA:

Cumulative kill probability	P_K =	.70
Target altitude	H =	35′
Target velocity	V_T =	-1000 fps
Target separation interval	Δt =	10 sec.
Number of attacking threats	N_T =	10

COMBAT SYSTEM DATA:

Missile/projectile velocity	V_M =	2000
fire control acquisition time	AT =	10 sec.
number of fire control radars	NFC =	1
search radar height	h =	90′
antenna rotation interval	ARI =	8 sec
weapon system minimum range	R_{min} =	4 nm.

FIND:

target detection range	DR =	18.3 nm
track range	TR =	15.67 nm.
velocity ratio	VR =	2/3
range of first intercept	IR_1 =	9.33 nm.
range of successive intercepts	IR_2 =	6.22 nm.
	IR_3 =	4.14 nm.

$IR_4 =$ 2.76

$IR_5 =$ 3.87

$IR_6 =$ 4.98

$IR_7 =$ 3.32

$IR_8 =$ 4.43

$IR_9 =$ 2.95

$IR_{10} =$ 4.06

$IR_{11} =$ NA

threat handling capacity $TC =$ 3

average target separation interval
for the next defense layer

before saturation $\Delta t_2 =$ 33.3 sec.

after saturation $\Delta t_2 =$ 14.3 sec

EQUATIONS:

radar horizon $D = 1.23 (\sqrt{H} + \sqrt{h})$

$\{D\} =$ nm. $\{H\}$ and $\{h\} =$ ft.

$$DR = D + (0.5 \times ARI \times V_T)$$

$$TR = DR + (V_T \times 2\ ARI)$$

$$VR = V_M / (V_M - V_T)$$

$$IR_1 = (TR + (AT \times V_T)) \times VR$$

$$IR_{n+1} = \{ IR_n + V_T (AT - \Delta t^*) \} \times VR$$

$$\Delta t^* = \Delta t / NFC$$

$$TC = n \text{ for } IR_{n+1} < R_{min}$$

$$IR_{L+i} = (IR_L + V_T (AT - i\Delta t^*)) \times VR$$

$$N_S = N_E (1 - P_K)$$

$$\Delta t_2 = (N_L + N_E)\Delta t_1 / (N_L + N_S)$$

CALCULATIONS:

V_T = -1000 fps = - 1/6 nm/sec.

Radar horizon;

D = 1.23(35 + 90) = 19 nm.

Detection range;

DR = 19 + (.5 x 8 x -1/6) = 18.33 nm.

Track range;

TR = 18.33 + (-1/6 x 2 x 8) = 15.66 nm.

Velocity ratio;

VR = (2000)/(2000 -(-1000)) = 2/3

Range of first intercept;

IR_1 = (15.66 + (10 x -1/6)) x 2/3 = 9.33 nm. Engaged

Subsequent intercept ranges;

IR_2 = (9.33 - 1/6(10 - 10)) x 2/3 = 6.22 nm. Engaged

IR_3 = (6.22 - 1/6(10 - 10)) x 2/3 = 4.14 nm. Engaged

IR_4 = (4.14 - 1/6(10 - 10)) x 2/3 = 2.76 nm. Leaker

IR_5 = (4.14 - 1/6(10 - 20)) x 2/3 = 3.87 nm. Leaker

IR_6 = (4.14 - 1/6(10 - 30)) x 2/3 = 4.98 nm. Engaged

IR_7 = (4.98 - 1/6(10 - 10)) x 2/3 = 3.32 nm. Leaker

IR_8 = (4.98 - 1/6(10 - 20)) x 2/3 = 4.43 nm. Engaged

IR_9 = (4.43 - 1/6(10 - 10)) x 2/3 = 2.95 nm. Leaker

IR_{10} = (4.43 - 1/6(10 - 20)) x 2/3 = 4.06 nm. Engaged

Threat capacity;

TC = 3

Threat interval for next layer;

Prior to saturation; N_E = 3 , N_L = 0

N_S = 3(1-.7) = .9

Δt_2 = (3 x 10)/.9 = 33.3 sec.

After saturation; N_E = 3 , N_L = 4

N_S = 3(1-.7) = .9

Δt_2 = (4 + 3)10 / (4 + .9) = 14.3 sec.

APPENDIX C

GUN SYSTEM VS. CRUISE MISSILE

This is a case study of a generic medium caliber gun system versus a generic sea skimming cruise missile. The object of the case study is to determine the lethality of the weapon system against the cruise missile and also to assess the contribution of the weapon system to the overall lethality of the associated combat system.

A. WARHEAD PARAMETERS

GIVEN: charge weight C = 8 lb. or density = —

metal weight M = 60 lb. or density = —

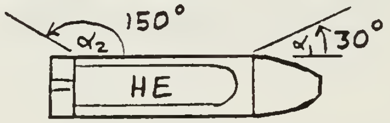
Guerney constant $\sqrt{2E}$ = 8800 or explosive: COMP. B

static spray angles α_1 = 30

α_2 = 150

fragment size = .5³ in³ and density = .283 lbs/in³

warhead geometry: (drawing or dimensions)



90% of case fragments

FIND: fragment weight in grains = 247

charge to mass ratio C/M = .133

fragment initial velocity V_0 = 3111

total number of fragments N = 1530

(N may be adjusted to reflect percentage of case that fragments)

Useful Equations:

1 pound = 7000 grains

Guerney formula $$V_0 = \sqrt{2E}\sqrt{\frac{C/M}{1+C/2M}}$$

CALCULATIONS:

Fragment weight;

$.5^3 \text{ in}^3 \times .283 \text{ lb/in}^3 \times 7000 \text{ gr/lb} = 247 \text{ grains}$

Charge to mass ratio;

$C/M = 8/60 = .1333$

Fragment initial velocity;

$V_o = 8800 \times .1333 / (1 + .1333/2) = 3111 \text{ fps.}$

Number of fragments;

$N = (.9 \times 60 \text{lbs.} \times 7000 \text{ gr/lb}) / 247 \text{ gr/frag} = 1530$

B. TARGET DESCRIPTION

GIVEN: target velocity V_T = -1000 fps

target drawing: including dimensions, component locations, and sizes.

see figure B.1

DETERMINE:

Presented area A_p = 28.5 ft^2

Minimum kill categories required:

(function of range and velocity)

KK < 10000 yds

Critical components for selected kill categories:

guidance, warhead, engine , fuel tank

ESTIMATE: component $P_{k/h}$ for the desired kill categories:

see table 2

CALCULATE: Vulnerable area from one or more aspects.

see table 2

EQUATIONS:

component vulnerable area;

$$A_v = A_p \times P_{k/h}$$

total vulnerable area;

$$A_V = \sum A_v$$

C. ENDGAME ANALYSIS

GIVEN:

WARHEAD PARAMETERS:

Early or late bird	=	EARLY	—
static warhead	α_1 =	30	
spray angles	α_2 =	150	
fragment number	N =	1500	
fragment velocity	V_o^* =	2800	

ENCOUNTER PARAMETERS:

Missile/Projectile velocity	V_M or V_P =	2000 fps	
missile approach angle	θ =	$-10°$	
target velocity	V_T =	-1000 fps	
target vulnerable areas A_V	top/side =	2.85	
	front/rear =	.18	

(* values may be adjusted to reflect average velocities)

FIND:

dynamic spray angles	γ_1 =	6	
	γ_2 =	60	
	ϕ_1 =	16	
	ϕ_2 =	70	
	γ_{cF} =	33	
range ratio	R/R_{DET} =	.80	
fragment spray densities			
as a function of R	$\rho(R)$ =	$385/R^2$	
as a function of R_{DET}	$\rho(R_{DET})$ =	$602/R_{DET}^2$	
projected vulnerable area	projected $A_V(\gamma_{cF})$ =	1.7	
Probability of a kill given a detonation	$P_{K/D}(\rho(R_{DET}), A_V(\gamma_{cF}))$ =	.47 (40.4')	

(calculate at various points as desired)

EQUATIONS:

$$\gamma_i = \arctan\left(\frac{V_M \sin(\theta) + V_o \sin(\theta + \alpha_i)}{V_M \cos(\theta) + V_o \cos(\theta + \alpha_i) - (V_T)}\right)$$

$$\phi_i = \gamma_i - \theta$$

$$\gamma_{cf} = (\gamma_1 + \gamma_2)/2$$

$$\frac{R}{R_{DET}} = \frac{|\vec{V}_{fa}|}{|\vec{V}_{f/T}|}$$

$$|\vec{V}_{fa}| = \sqrt{(V_M + V_o \cos\alpha_{cf})^2 + (V_o \sin\alpha_{cf})^2}$$

$$|\vec{V}_{f/T}| = \sqrt{(V_{fa} \cos(\gamma_{cf}) - V_T)^2 + (V_{fa} \sin\gamma_{cf})^2}$$

$$\rho(R) = \frac{N}{2\pi \times R^2 \times (\cos(\phi_1) - \cos(\phi_2))}$$

$$\text{Projected } A_V = A_{V_1} |\sin(\gamma_{cf})| + A_{V_2} |\cos(\gamma_{cf})|$$

$$P_{K/D} = 1 - \exp(-\rho \times \text{projected } A_V)$$

CALCULATIONS:

EARLY SHELL

Dynamic spray angles:

$$\gamma_1 = \arctan\left(\frac{2000\ \sin(-10) + 7000\ \sin(-10 + 30)}{2000\ \cos(-10) + 7000\ \cos(-10 + 30) - (-1000)}\right)$$

$$= 6$$

$$\gamma_2 = \arctan\left(\frac{2000\ \sin(-10) + 7000\ \sin(-10 + 150)}{2000\ \cos(-10) + 7000\ \cos(-10 + 150) - (-1000)}\right)$$

$$= +60$$

$$\gamma_{cF} = (6 + 60)/2 = 33$$

$$\phi_1 = 6 - (-10) = 16$$

$$\phi_2 = 60 - (-10) = 70$$

$$\alpha_{cF} = (30 + 150)/2 = 90$$

R/R_{DET} :

$$|\vec{V}_{fa}| = \sqrt{(2000 + 2800\ \cos(90))^2 + (2800\ \sin(90))^2} = 3441$$

$$|\vec{V}_{f/T}| = \sqrt{(3441\ \cos(33) - (-1000))^2 + (3441\ \sin(33))^2} = 4314$$

$$R/R_{DET} = 3441/4314 = .80$$

$$\rho(R) = \frac{1500}{2\pi R^2\ (\cos(16) - \cos(70))} = \frac{385}{R^2}$$

$$\rho(R_{DET}) = \frac{385}{R^2_{DET}\ (0.80)^2} = \frac{602}{R^2_{DET}}$$

Projected A_V:

$$A_V\ (\gamma_{cF}) = 2.85\,|\sin(33)| + .18\,|\cos(33)| = 1.70$$

$P_{K/D}$:

First find where fragment spray covers the target.

Knowing two angles, $\gamma_1 = 6$ and $(\gamma_2 - \gamma_1)/2 = 27$

and one side, side = target length / 2 = 10′

R_{DET} = 2.3 ft. to cover target

$P_{K/D}$ at 2.3 ft. = $1 - \exp((-602/2.3^2) \times 1.70) = 1.0$

$P_{K/D}$ at 40.4 ft. = $1 - \exp((-602/40.4^2) \times 1.70) = .47$

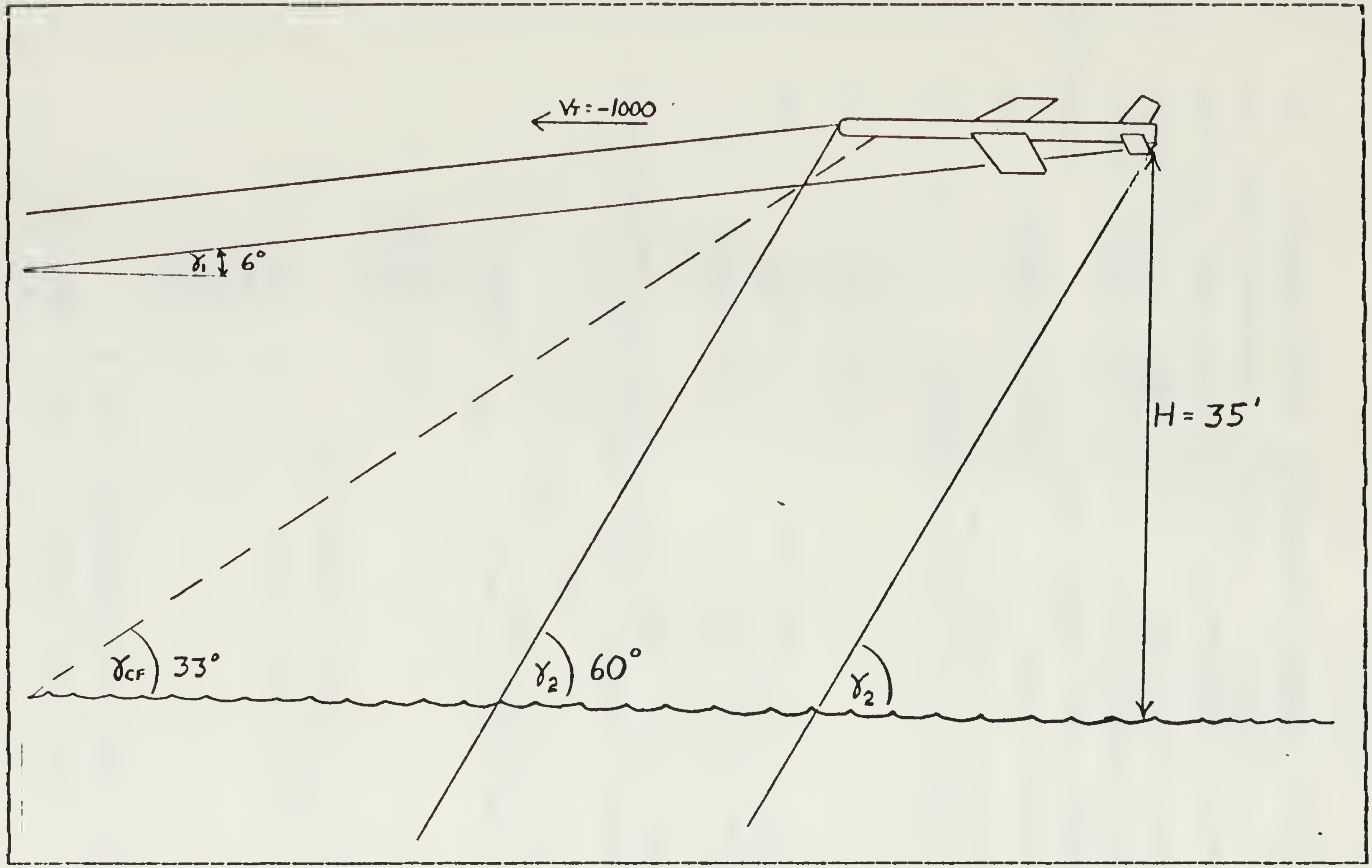


Figure C.1   Early Shell Spray Zones

D. DETERMINATION OF WEAPON SYSTEM LETHALITY

GIVEN:

Circle of Equal Probability CEP = 3 mils

or

standard deviation = —

fuzing probability P_f = 0.6

fuze cutoff range r_c = 60′

fragment spray density as a function of R_{DET} $\rho(R_{DET}) = 602/R_{DET}^2$

projected vulnerable area $A_V(\gamma_{CF})$

for desired kill levels $A_V(\gamma_{CF})$ = 1.7 KK

= NA K

= NA A

= NA B

target altitude H = 35′

center spray angle γ_{CF} = 33

Firing doctrine:

open fire at 10000 yds. firing 15 rounds/min

cease fire when target destroyed or < 2000 yds.

FIND:

lethal radius (r_ℓ)

for desired kill levels r_ℓ = 38.4 KK

= NA K

= NA A

= NA B

Carlton scaling parameter (r_ℓ)

for desired kill levels r_o = 46 KK

= NA K

= NA A

= NA B

single shot kill probability (P_{KSS})

for desired kill levels P_{KSS} = TABLE 3 KK

= NA K

P_{Kss} = NA A

= NA B

fuze cutoff correction factor = Table 3

sea skimmer correction factor = NA ($h > r_c$)

corrected kill probability P_{Kss} = Table 3

cumulative kill probability

(weapon system lethality) P_K = .948

EQUATIONS:

$$CEP = 1.177\,\sigma_r$$

$$P_{K/D} = 0.5 = 1 - \exp(-\rho(r_o)\,A_V)$$

$$P_{Kss} = P_f\left(\frac{r_o^2}{r_o^2 + 2(\sigma_r)^2}\right)$$

$$\text{fuze cutoff factor} = (1 - \exp(C_2\, r_c^2)$$

$$C_2 = \left(-\frac{r_o^2 + 2\sigma_r^2}{2\,\sigma_r^2\, r_o^2}\right)$$

sea skimmer correction factor;

$$1 - \frac{1}{2\pi}\int_{\pi}^{2\pi} \exp\left(\frac{C_2\, h^2}{\sin^2(2\pi - \theta)}\right) d\theta$$

$$h = H / \sin(\gamma_{cF})$$

$$P = 1 - \prod_{i=1}^{n}(1 - P_{Kss_i})$$

CALCULATIONS:

Lethal radius;

$.5 = \exp(-\rho A_V)$

$\ln(.5) = (-602/R_{DET}^2) \times 1.7$

$R_{DET}^2 = 1476 \implies R_{DET}$ for $P_{K/D}$ of $.5 = 38.4 = r_\ell$

Carlton scaling parameter;

$r_o = 1.2\, r_\ell = 46$ ft.

Calculation of P_{KSS} ;

$$P_{KSS} = \left(\frac{46^2}{46^2 + 2(CEP/1.177)^2}\right) \times 0.6$$

CEP must be determined at the various intercept ranges in order to calculate P_{KSS}.

For 3 mil CEP

CEP = 3yds/thousand yds of range

= 9 ft / (intercept range/1000)

Intercept range (IR)

IR = firing range (FR) x VR

$VR = V_M / (V_M - V_T) = 2/3$

Firing range; (fires every 60/15 = 4 sec)

$FR = 10000 \text{ yds} + (i \times V_T \times 4) \quad i = 0, 1, 2, \ldots$

Fuze cutoff factor;

$$1 - \exp\left[-\left(\frac{2(CEP/1.177)^2 + r_o^2}{2(CEP/1.177)^2\, r_o^2}\right) r_c^2\right]$$

TABLE 3

Gun System P_{KSS} and P_K

Round #	FR	IR	CEP	P_{KSS}	Fuze C/O	Corr. P_{KSS}	Cum. P_K
1	10000	6667	60	.174	.91	.158	.158
2	8668	5781	52	.211	.93	.196	.323
3	7336	4893	44	.259	.95	.246	.489
4	6004	4004	36	.319	.97	.309	.647
5	4672	3116	28	.391	.99	.387	.784
6	3340	2227	20	.472	1.0	.472	.886
7	2008	1339	12	.546	1.0	.546	.948

E. DETERMINATION OF COMBAT SYSTEM LETHALITY

GIVEN:

TARGET DATA:

Cumulative kill probability	P_K =	.948
Target altitude	H =	35′
Target velocity	V_T =	-1000 fps.
Target separation interval	Δt_2 =	33.3 Before sat. 14.3 After sat.
Number of attacking threats	N_T =	10

COMBAT SYSTEM DATA:

Missile/projectile velocity	V_P =	2000 fps
fire control acquisition time	AT =	9 sec.
number of fire control radars	NFC =	1
search radar height	h =	90′
antenna rotation interval	ARI =	8 sec
weapon system minimum range	R_{min} =	1000 yds

FIND:

target detection range	DR =	NA
track range	TR =	NA
velocity ratio	VR =	2/3
range of first intercept	IR_1 =	6667 yds.
range of successive intercepts	IR_2 =	Table 4
	IR_3 =	"

$IR_4 =$ Table 4

$IR_5 =$ "

$IR_6 =$ "

$IR_7 =$ "

$IR_8 =$ "

$IR_9 =$ "

$IR_{10} =$ "

$IR_{11} =$ NA

threat handling capacity $TC = \infty$

average target separation interval
for the next defense layer

before saturation $\Delta t_3 =$ NA

after saturation $\Delta t_3 =$ NA

EQUATIONS:

radar horizon $D = 1.23\ (\sqrt{H} + \sqrt{h})$

$\{D\} =$ nm. $\{H\}$ and $\{h\} =$ ft.

$DR = D - (0.5 \times ARI \times V_T)$

$TR = DR - (V_T \times 2\ ARI)$

$VR = V_M / (V_M - V_T)$

$IR_1 = (TR + (AT \times V_T)) \times VR$

$IR_{n+1} = \{ IR_n + V_T(AT - \Delta t^*) \} \times VR$

$\Delta t^* = \Delta t / NFC$

$TC = n$ for $IR_{n+1} < R_{min}$

$IR_{L+i} = (IR_L + V_T(AT - i\Delta t^*)) \times VR$

$N_S = N_E\ (1 - P_K)$

$\Delta t_2 = (N_L + N_E)\Delta t_1 / (N_L + N_S)$

CALCULATIONS:

Since the gun system fires continuously at one target until it is destroyed or reaches minimum range, the total engagement time is a function of acquisition time and the time required between the initial and the final shot. Thus

$$ET = AT + (N_{SHOTS} - 1)(60 / \text{firing rate})$$
$$= 9 + (N_{SHOTS} - 1)(60/15)$$

The limiting engagement time is nine seconds, which means the system could engage a different target every nine seconds with one shot.

The first target to be engaged by the gun system will have the maximum cumulative P_K. The cumulative P_K for subsequent targets will depend on the target separation interval. For subsequent targets, assuming the final shot against the first target is fired at the minimum firing range of 2000 yards, the relationship between target separation interval, number of shots fired, and cumulative P_K are calculated using the data of table 3. Table 4 shows the results.

For the mean threat interval prior to missile system saturation, t = 33.3 seconds, table 4 shows that the gun system would be able to engage a surviving target with the maximum cumulative P_K of 0.948. Following missile system saturation the average threat interval is reduced to 14.3 seconds. From table 4, the gun system can engage subsequent targets at that inteval with a P_K of 0.76. From Appendix B, actual target threat intervals after saturation will be either ten or twenty seconds. For the ten second intervals the gun system P_K is 0.546, and for the twenty second intervals the P_K is 0.853.

The threat capacity of the gun system is

$$TC = 1 \quad \text{for } \Delta t^* < 9 \text{ sec.}$$
$$= \infty \quad \text{with reduced } P_K \text{ for } 9 < \Delta t^* < 33 \text{ sec.}$$
$$= \infty \quad \text{with } P_K = .948 \text{ for } \Delta t^* > 33 \text{ sec.}$$

TABLE 4

Gun System P_K for Subsequent Targets

TARGET Δt	SHOTS FIRED	INIT. FR	INIT IR	CUM. P_K	CUM. P_S
<9	0	-	-	0	1
9	1	2008	1339	.546	.454
13	2	3340	2227	.760	.240
17	3	4672	3116	.853	.147
21	4	6004	4004	.898	.102
25	5	7336	4893	.923	.077
29	6	8668	5781	.939	.061
33	7	10000	6667	.948	.052

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